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A shear-lag model with a cohesive fibre–matrix interface for analysis of fibre pull-out

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ABSTRACT

A shear-lag model with a cohesive fibre–matrix interface has been developed for the analysis of stress transfer between the fibre and the matrix in fibre-reinforced composites in this paper. A bilinear cohesive damage evolution law is used to describe the fibre–matrix interface behaviour. The governing equations for the interfacial shear stress and the axial stress in the fibre are derived. Accurate analytical solutions are obtained when the fibre–matrix interface is in the initial linear elastic deformation regime. When debonding occurs, interfacial damage and softening are modelled by superposing two elastic stress systems and satisfying the damage evolution law at both ends of the damage process zone, and approximate analytical solutions are obtained. The stress distribution and evolution during the fibre pull-out, the maximum pull-out force and the pull-out curve have been analysed using a shear strength-based debonding criterion. Analytical expressions for the maximum fibre pull-out force and its limit as the embedded fibre length approaches infinity are obtained. In addition, the new function proposed for describing the radial distribution of the shear stress in the matrix fixes the problem of zero shear-lag parameter when b/a approaches infinity, enabling the shear-lag analysis to deal with low fibre volume fractions. Generally, the analytical solutions compare satisfactorily well to the cohesive finite element calculations and experimental data in the literature.

1. Introduction

The load transfer mechanism between the fibre and the matrix and the fibre–matrix interface behaviour play an important role in determining the mechanical properties of fibre-reinforced composites such as elastic modulus, tensile strength and fracture toughness, and have received considerable attention and extensive investigations. Tensile stresses acting on the composites can be transferred between the matrix and fibres by shear at the fibre–matrix interface. Theoretical analysis of the load transfer and interfacial debonding problem during fibre pull-out can be classified into two principal approaches; one is the strength-based approach where the interfacial debonding takes place when the interfacial shear stress reaches the interfacial strength (Cox, 1952; Hsueh, 1988, 1992; Landis and McMeeking, 1999; Lawrence, 1972; Leung and Li, 1991; McCartney, 1992; Nairn, 1997, 2004; Nayfeh, 1977), and the other is the fracture mechanics-based approach where the interfacial debonding is treated as a mode II fracture which propagates once the interfacial toughness is overcome (Budiansky et al., 1986; Gao et al., 1988; Gurney and Hunt, 1967; Hutchinson and Jensen, 1990; Nairn, 2000; Stang and Shah, 1986; Zhou et al., 1992). The two theories of interfacial debonding and fibre pull-out have been compared experimentally (Kim et al., 1992; Zhandarov et al., 2001),

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and theoretically (Leung and Li, 1991; Stang et al., 1990), indicating the conditions of their validity.

In the strength-based theories, the original shear-lag theory (Cox, 1952) has been widely adopted to obtain the shear stress distribution at the fibre–matrix interface. Modifications of the classical shear-lag model have been made to obtain improved results (Hsueh, 1992; McCartney, 1989; Nairn, 1997, 2004; Nayfeh and Abdelrahman, 1998). Extensions of the shear-lag model with introducing interfacial friction have been proposed to analyse interfacial debonding during the fibre pull-out or push-out (Budiansky et al., 1986; Leung and Li, 1991; McCartney, 1989; Shetty, 1988; Zhou et al., 1993).

Recently, owing to their extraordinary physical and mechanical properties such as high tensile stiffness and strength and low density, carbon nanotubes (CNTs) find promising applications as reinforcements in advanced structural composites. Early experimental measurements showed very disappointing improvements in the mechanical properties of carbon nanotube based composites. It has been identified through both experimental and numerical studies that the CNT-matrix interfacial characteristics (interfacial strength and length) critically control the performance of such composites and therefore they have attracted considerable attention of many researchers. Various interfacial interaction models have been adopted in analysis of the stress transfer between carbon nanotube and matrix. For examples, the van der Waals interfacial interaction between the polymer matrix and the CNT and the interfacial chemical bonding have been modelled with the Lennard-Jones potential, and the many-body bond-order potential, respectively, in analysis of the shear strength of carbon nanotube-polymer matrix interfaces (Chen et al., 2010; Frankland et al., 2002, 2003). Nonlinear cohesive laws for CNT-polymer matrix interfaces have been established based on the van der Waals interfacial interaction (Jiang et al., 2006; Lu et al., 2008) and chemical bonding (Jiang, 2010), respectively. The classical shear-lag model has been adopted to predict the interfacial stress transfer in CNT-reinforced polymer composites (Gao and Li, 2005), to analyse the carbon nanotube pull-out from a polymer matrix (Frankland and Harik, 2003), and to investigate fracture toughness enhancement with introducing a linear interface law to account for CNT-matrix interfacial bond breaking (Chen et al., 2010). The shear-lag model has also been used to analyse the pull-out test of CNT-coated carbon fibres in a polyester matrix (Agnihotri et al., 2012), and cohesive zone finite element models have been established with introducing a non-linear interface cohesive law to model the pull-out response of CNT-coated fibres (Agnihotri et al., 2012; Jia et al., 2014).

The development of cohesive laws for characterising the CNT-matrix interfacial properties, and the application of different cohesive laws together with the classical shear-lag theory in analysing the load transfer between CNT and matrix inspire us to establish an analytical relationship between the load transfer and evolution and the interfacial cohesive properties. We first derive the governing equations for a shear-lag model with a cohesive fibre–matrix interface by introducing an interfacial bilinear cohesive law into the classical shear-lag model in Section 2. The accurate analytical solution for the cohesive interface in the initial linear elastic deformation regime is given in Section 3. In Section 4, we propose a simple method to obtain approximate analytical solutions when interfacial damage and softening occur. The analytical solutions for the distribution and evolution of the interfacial shear stress and axial stress in the fibre are compared to the cohesive zone finite element calculations and experimental data available in the literature.

2. A shear-lag model with a cohesive fibre–matrix interface

A composite cylinder shear-lag model is adopted for the analysis of the load transfer from the fibre to the matrix when the fibre is loaded. As shown in Fig. 1, a single fibre with a radius a is embedded with a length L in a coaxial matrix cylinder with an outer radius b. The fibre is subjected to an axial tensile stress σ_p at the loaded end $(z = L)$. The embedded end face $(z = 0)$ between the fibre and matrix can be perfectly bonded or completely free. It is assumed that both the fibre and matrix are elastic, and the interface transfers stresses between the fibre and matrix by interfacial shear. The outer surface $(r = b)$ is stress-free.

The interface $(r = a)$ between the fibre and matrix is modelled as a cohesive interface in pure shear mode. The shear behaviour of the cohesive interface is described by a bilinear cohesive traction-separation law, as shown in Fig. 2. The cohesive law is characterised by an initial linear elastic regime followed by a linear softening regime. Interfacial damage initiates once the interfacial shear stress reaches the cohesive strength τ_0 and the shear separation reaches the critical value δ_0. Beyond this point, as the shear separation increases further, the shear stress decreases due to material degradation until the complete failure (interfacial debonding) begins where the separation reaches the critical value δ_1 and the traction or cohesive strength acting across the cohesive interface is reduced to zero.

The bilinear cohesive traction-separation constitutive relation is given by

$$\tau = \begin{cases} K_0\delta & \text{if } 0 \leqslant \delta^{max} \leqslant \delta_0 \\ (1-D)K_0\delta & \text{if } \delta_0 \leqslant \delta^{max} \leqslant \delta_1 \\ 0 & \text{if } \delta^{max} \geqslant \delta_1 \end{cases} \tag{1}$$

where K_0 is the initial shear stiffness of the cohesive interface, δ^{max} is the maximum value of the shear separation attained during the fibre pull-out, and D is the scalar damage variable.

This law assumes that the cohesive surfaces initially are intact without any relative displacement, and exhibit reversible linear elastic behaviour until the traction reaches the cohesive strength τ_0 or equivalently the separation exceeds δ_0. Beyond δ_0, the traction reduces linearly to zero up to δ_1 and any unloading takes place irreversibly. The area under the traction-separation curve represents the fracture energy, G_c, of the cohesive crack.