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Hardening effects on strain localization predictions in porous ductile materials using the bifurcation approach

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ABSTRACT

The localization of deformation into planar bands is often considered as the ultimate stage of strain prior to ductile fracture. In this study, ductility limits of metallic materials are predicted using the Gurson–Tvergaard–Needleman (GTN) damage model combined with the bifurcation approach. Both the GTN constitutive equations and the Rice bifurcation criterion are implemented into the finite element (FE) code ABAQUS/Standard within the framework of large plastic strains and a fully three-dimensional formulation. The current contribution focuses on the effect of strain hardening on ductility limit predictions. It is shown that the choice of void nucleation mechanism has an important influence on the sensitivity of the predicted ductility limits to strain hardening. When strain-controlled nucleation is considered, varying the hardening parameters of the fully dense matrix material has no effect on the porosity evolution and, consequently, very small impact on the predicted ductility limits. For stress-controlled nucleation, the porosity evolution is directly affected by the strain hardening characteristics, which induce a significant effect on the predicted ductility limits. This paper also discusses the use of a micromechanics-based calibration for the GTN *q*-parameters in the case of strain-controlled nucleation, which is also shown to allow accounting for the hardening effects on plastic strain localization.

1. Introduction

In sheet metal forming processes, strain localization occurrence is one of the main defects that limit the formability of stretched sheet metals. A practical approach to characterize the formability of sheet metals is the use of the so-called forming limit diagram (FLD), which consists of a plot of the in-plane critical strains at the onset of localized necking (see, e.g., Keeler and Backofen, 1963; Goodwin, 1968). Many literature works have been devoted to the development of theoretical strain localization criteria for the FLD prediction. Two types of necking are usually encountered in stretched metal sheets: diffuse necking and localized necking. In his pioneering work, Considère (1885) proposed a diffuse necking criterion based on the maximum force principle in the particular case of uniaxial tension, while Swift (1952) extended Considère's criterion to the case of in-plane biaxial loading. Alternatively to these two earlier diffuse necking criteria, Hill (1952) developed a different approach for localized necking prediction based on bifurcation theory. This latter criterion indicates that the localized band emerges along a direction of zero extension. However, no bifurcation is predicted with this criterion in the expansion domain of the FLD. To overcome this limitation, Hill's localized necking criterion may be

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combined with Swift's diffuse necking criterion to determine a complete FLD of a sheet metal. Another approach for localized necking prediction, which is referred to as the M–K criterion, assumes the existence of an initial geometric imperfection (Marciniak and Kuczynski, 1967) or an initial material imperfection (see, e.g., Yamamoto, 1978) in the sheet metal. This M–K approach is one of the most commonly used criteria for the determination of FLDs, due to its simple use and its applicability to a wide range of constitutive models. Note also that Hora et al. (1996) proposed a modification to Considère's maximum force criterion to take the strain-path dependency into account in the prediction of localized necking. Other theoretical criteria based on bifurcation theory have been developed in the literature during the last few decades (see, e.g., Abed-Meraim et al. (2014) for a detailed discussion and comparison). In this regard, Hill (1958) developed a general bifurcation (GB) criterion based on the loss of uniqueness for the solution of the associated boundary value problem. This GB criterion, written in a local framework, allows predicting the onset of diffuse necking in sheet metals. A condition less conservative than the BG criterion has been proposed by Valanis (1989), and corresponds to the stationarity of the stress state. Analogous approaches based on bifurcation theory have been established in Rudnicki and Rice (1975), Stören and Rice (1975) and Rice (1976) to predict localized necking or shear band instabilities in solid materials. In these criteria, the occurrence of localization bifurcation is related to the loss of ellipticity of the associated boundary value problem.

The above criteria are generally coupled with constitutive models for the prediction of critical strains. In Rudnicki and Rice (1975) and Rice (1976), it has been shown that, in the framework of phenomenological constitutive models with associated plasticity and smooth yield surface, localization bifurcation cannot be predicted in the positive hardening regime. In such a situation, damage-induced softening is required in the constitutive model. In the literature, damage models may be classified into two main theories. The first is the continuum damage mechanics approach (see, e.g., Lemaitre, 1985, 1992; Kachanov, 1986), where damage is described by a phenomenological variable, which may be isotropic scalar or anisotropic tensor-valued, representing a surface density of defects. The second theory is based on micromechanical analysis of void growth, which describes the complex ductile damage mechanisms in porous materials. In this regard, Gurson (1977) proposed a void growth model that takes into consideration the effect of hydrostatic stress on porous materials. Later, several modifications of the Gurson model have been made to account for void nucleation, coalescence, hardening of the dense matrix, and plastic anisotropy (see, e.g., Chu and Needleman, 1980; Tvergaard, 1981, 1982a, 1982b; Tvergaard and Needleman, 1984; Benzerga and Besson, 2001). In the so-called Gurson–Tvergaard–Needleman (GTN) model, the void volume fraction acts as a damage internal variable responsible for the progressive loss of load carrying capacity. In the present work, the GTN model with isotropic hardening, for the fully dense matrix material, and a von Mises yield criterion is considered to describe ductile damage in porous materials.

More specifically, in the current contribution, an approach that combines the GTN model with the Rice bifurcation criterion is proposed to predict ductility limits of porous materials. To this end, the resulting coupling is implemented into the finite element (FE) code ABAQUS/Standard in the framework of large plastic strains and a fully three-dimensional formulation. In a recent work, a similar approach has been followed (see Mansouri et al., 2014) to investigate the effects of hardening and damage parameters on the prediction of ductility limits of porous materials. In this latter study, it has been shown that the damage parameters have a significant effect on the ductility limits for all of the strain paths considered, while the effect of strain hardening was only perceptible for the plane strain tension loading path. These results, which have been found in the case of strain-controlled nucleation, are explained by the fact that, in such nucleation modeling, the porosity evolution is totally governed by the damage parameters. To reproduce the effect of strain hardening of the dense matrix on the ductility limits, as classically observed in the literature (see, e.g., Yamamoto, 1978; Hora et al., 1996; Zhao et al., 1996), a micromechanics-based calibration for the GTN q-parameters is adopted in the current study, as suggested by Faleskog et al. (1998), in order to account for the strain hardening effect on the porosity evolution. Alternatively, for the same purpose, stress-controlled nucleation is also considered in this study within the GTN model (see, e.g., Needleman and Rice, 1978; Chu and Needleman, 1980; Saje et al., 1982), which allows the strain hardening effect on the porosity evolution to be accounted for. In this latter case, the normality of the plastic flow rule does not hold, which induces a destabilizing effect that promotes strain localization.

2. Porous elastic–plastic constitutive equations

The so-called GTN damage model, which is an extension of the original Gurson model (1977), is briefly described in this section. Only the main constitutive equations are recalled; the full details can be found in Tvergaard (1982c) and Tvergaard and Needleman (1984).

2.1. Large strain kinematics

Within the large strain framework, the material behavior is commonly described by rate constitutive equations and, to achieve material objectivity, objective rates must be used. To this end, the constitutive models are often written in a convenient frame in order to simplify their formulation and further their FE implementation. The large deformation theory used here, which is also consistent with that adopted in the FE code ABAQUS, is recalled hereafter.

The kinematics of large elastic–plastic deformation are based on the multiplicative decomposition of the deformation gradient $\mathbf{F}$ into a plastic part $\mathbf{F}^p$ and an elastic part $\mathbf{F}^e$

$$\mathbf{F} = \mathbf{F}^e \cdot \mathbf{F}^p, \quad \mathbf{F}^e \cong (\mathbf{1} + \boldsymbol{\varepsilon}^e) \cdot \mathbf{R}. \tag{1}$$

In Eq. (1), the elastic strains are considered to be small with respect to unity, which is a reasonable assumption for sheet metals; nevertheless, large rotations are rigorously