Mechanics of Materials 91 (2015) 226–240

Contents lists available at ScienceDirect

Mechanics of Materials

journal homepage: www.elsevier.com/locate/mechmat

Effective elastic properties of periodic hexagonal honeycombs

Sardar Malek, Lorna Gibson *

Department of Materials Science and Engineering, Massachusetts Institute of Technology, 77 Massachusetts Ave, Cambridge, MA 02139, USA

ARTICLE INFO

Article history:
Received 11 February 2015
Received in revised form 1 July 2015
Available online 17 July 2015

Keywords:
Honeycomb
Homogenization
Effective properties
Modeling
Finite element method
Unit cell

ABSTRACT

We investigate the elastic behavior of periodic hexagonal honeycombs over a wide range of relative densities and cell geometries, using both analytical and numerical approaches. Previous modeling approaches are reviewed and their limitations identified. More accurate estimates of all nine elastic constants are obtained by modifying the analysis of Gibson and Ashby (1997) to account for the nodes at the intersection of the vertical and inclined members. The effect of the nodes becomes significant at high relative densities. We then compare the new analytical equations with previous analytical models, with a numerical analysis based on a computational homogenization technique and with data for rubber honeycombs over a wide range of relative densities and cell geometries. The comparisons show that both the new analytical equations and numerical solutions give a remarkably good description of the data. The results provide new insights into understanding the mechanics of honeycombs and designing new cellular materials in the future.

1. Introduction

Honeycombs are widely used as the cores of sandwich panels in the aerospace, automotive, construction, marine and wind energy industries. The separation of stiff, strong faces (for instance, fiber reinforced composite laminates) by a lightweight, typically cellular core, increases the moment of inertia of the panels with little increase in weight, making them efficient for resisting bending and buckling loads.

Honeycombs are used extensively in the aerospace industry, due to their high specific stiffness and strength (Burton and Noor, 1997). The response of a sandwich structure under different types of load depends, in part, on the effective (equivalent) properties of the core (Allen, 1969; Burton and Noor, 1997; Hohe and Becker, 2001; Balawi and Abot, 2008). As a result, the effective elastic and inelastic properties of honeycombs have been widely studied using analytical and numerical models (e.g. Kelsey et al., 1958; Gibson et al., 1982; Gibson and Ashby, 1988, 1997; Zhang and Ashby, 1992; Burton and Noor, 1997; Simone and Gibson, 1998; Balawi and Abot, 2008). Many of these models use approximations that limit their validity to low values of the ratio of cell wall thickness to length and to isotropic cell walls. With a view towards applying honeycomb models to natural materials such as balsa wood, with relatively high values of cell wall thickness to length (compared with engineering honeycombs) and with anisotropic cell walls, we extend Gibson and Ashby's honeycomb model.

We first review some widely used models available in the literature and identify the limitations of these models in Section 2. The analysis of Gibson and Ashby (1997) for hexagonal honeycombs with constant cell wall thickness is extended in Section 3 to allow more accurate estimates of all nine elastic constants of honeycombs with cell walls of constant thickness. In addition, new analytical equations for the elastic constants of honeycombs with double thickness vertical cell walls are presented in Appendix A.

A 3D numerical analysis for the elastic constants, based on a computational homogenization technique, is described in Section 4. It is intended that the numerical model provide

* Corresponding author.
E-mail address: ljgibson@mit.edu (L. Gibson).

http://dx.doi.org/10.1016/j.mechmat.2015.07.008

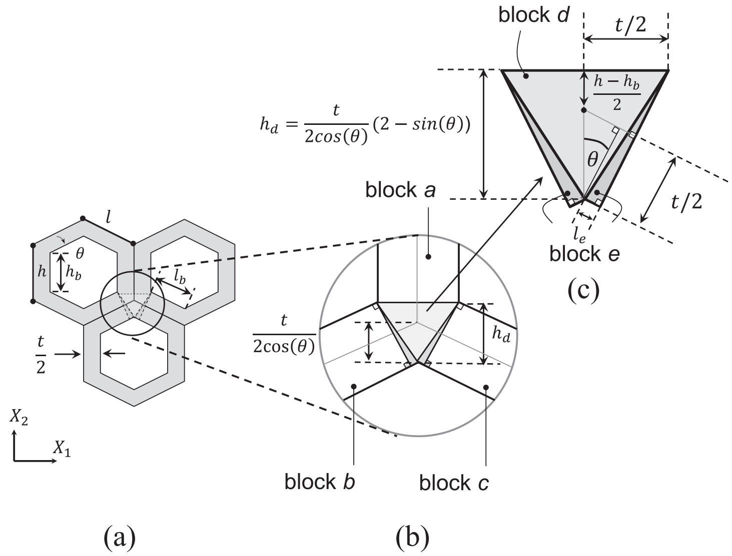

Fig. 1. Effective bending lengths of the cell wall, l_b and h_b, for periodic hexagonal honeycombs with uniform wall thickness. Schematic representation of (a) honeycomb cells, (b) geometrical dimensions and (c) building blocks of the node region.

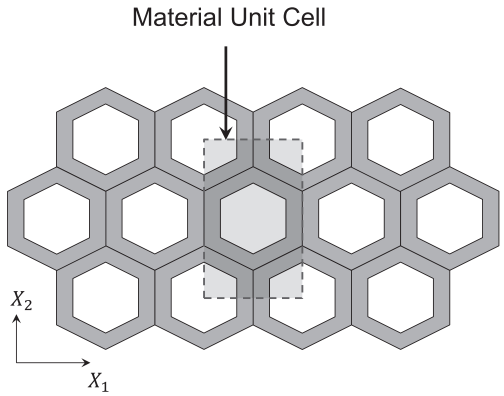

Fig. 2. Schematic representation of periodic, hexagonal honeycombs. A unit cell of the material has been highlighted.

reference solutions to examine the range of validity of the new analytical equations. In addition, both the analytical and numerical results are compared with experimental data on rubber honeycombs (Gibson, 1981) in Section 5.

Both the new analytical results and the numerical results give a remarkably good description of the available experimental data. The numerical approach can easily be adapted for an orthotropic cell wall, as in the case of woods. Finally, the use of the numerical approach in investigating the effect of relative density and cell geometry on the effective elastic properties of honeycombs and understanding the mechanics of honeycombs is discussed in Section 6.

2. Literature

One of the most widely used models for the elastic constants of honeycombs is that of Gibson and Ashby and co-workers (Gibson et al., 1982; Gibson and Ashby, 1988, 1997). Their initial studies of the in-plane moduli focussed on bending deformation in cell walls of uniform thickness; bending-based models give a good description of the elastic moduli of low relative density honeycombs. They also modeled the out-of-plane elastic moduli. In general, good agreement between the models and experimental results was reported for all properties, although some discrepancies were observed, especially for in-plane shear properties which were attributed to shear deformation in the testing jig. Later on, they extended the models to include the contribution of axial and shear deformations to the in-plane moduli and to include honeycombs with vertical walls of double thickness, representative of many honeycombs made by the corrugation and expansion processes (Gibson and Ashby, 1997).

Honeycombs with double thickness vertical cell walls have attracted more attention in the literature than those with constant cell wall thickness. Masters and Evans (1997) derived analytical equations for the effective in-plane elastic properties of honeycombs with double thickness vertical cell walls, considering axial, bending and shear deformations as well as hinging (change of the angle at the intersection of vertical and inclined walls), obtaining results similar to those of Gibson and Ashby (Gibson and Ashby, 1997). The full set of analytical equations for honeycombs were also given by Burton and Noor (1997) by incorporating a coefficient in the equations derived by Gibson and Ashby (1988).

There are several limitations to the above models. An exact expression for one of the out-of-plane shear moduli is not obtained; only upper and lower bounds. The accuracy of the in-plane models accounting for shear and axial deformation of the cell walls was never fully investigated. Although the analytical models given by Gibson and Ashby (1997) and Masters and Evans (1997) are similar and predict the effective elastic properties of low relative density honeycombs well, they become less accurate with increas-