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Constitutive relations for wave propagation in a double porosity solids

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ABSTRACT

This study considers the propagation of harmonic plane waves in a double porosity solid saturated with a non-viscous fluid. Existence of three longitudinal waves and a transverse wave is explained through the Christoffel equations, which define the phase velocities and the polarizations of constituent particles. Reflection of plane waves is studied at the stress-free plane surface of the composite medium. A numerical example is solved to calculate the partition of incident energy among the reflected waves. Conservation of the incident energy could be achieved only through the share of interaction energy. The presence of interaction energy is something unexpected, when the medium behaves non-dissipative to the propagation of elastic waves. The reason lies in the constitutive relations being used for double porosity medium, which are not symmetric in elastic coupling, as required by the Betti's reciprocal theorem. Corrections to the constitutive relations are proposed and the correct relations are used to study the reflection and refraction phenomena at the boundaries of double porosity solids. Dispersion in velocity and attenuation is studied for the four attenuated waves in double porosity solid saturated with viscous fluid.

1. Introduction

A fundamental revision of conventional fluid-rock deformation finds favor in almost all solid-earth geosciences, including hydrology, seismic exploration and earthquake forecasting. Because, most near-surface rock-masses are fractured to some degree, it is natural to examine the coupled fluid-rock deformation through the double porosity network. To study the fluid flow in hydrocarbon reservoirs and aquifers, the double-porosity medium is perhaps the simplest and ideal model for crustal rocks (Barenblatt et al., 1960; Warren and Root, 1963). This model considers a fracture network that divides the porous matrix into different blocks and the fluid in fractures surrounds the disaggregated matrix blocks supported entirely by fluid pressure. This requires extending the formulation of Biot (1956, 1962) to a composite medium containing fracture network in porous matrix. There have been many attempts to incorporate microcracks into the rock models that account for partial saturation effects and the fluid-flow during the passage of seismic waves (Dvorkin and Nur, 1993; Thomsen, 1995). In these attempts, the approach has been limited mainly to modify the elastic parameters of Biot's theory for the introduction of microcracks.

To reflect the effect of state and constitution of inter-diffusing materials, Aifantis (1980a) formulated the constitutive equations for the stress and the diffusive force. Analogous to this double-diffusivity model, Aifantis (1980b) considered the problem of flow in media with double porosity (e.g., fissured rocks) by utilizing the concept of "multiporosity" and the continuum theory of mixtures. Very recently, Kalambakas and Aifantis (2012) applied the graph theory concepts to the double-diffusivity theory

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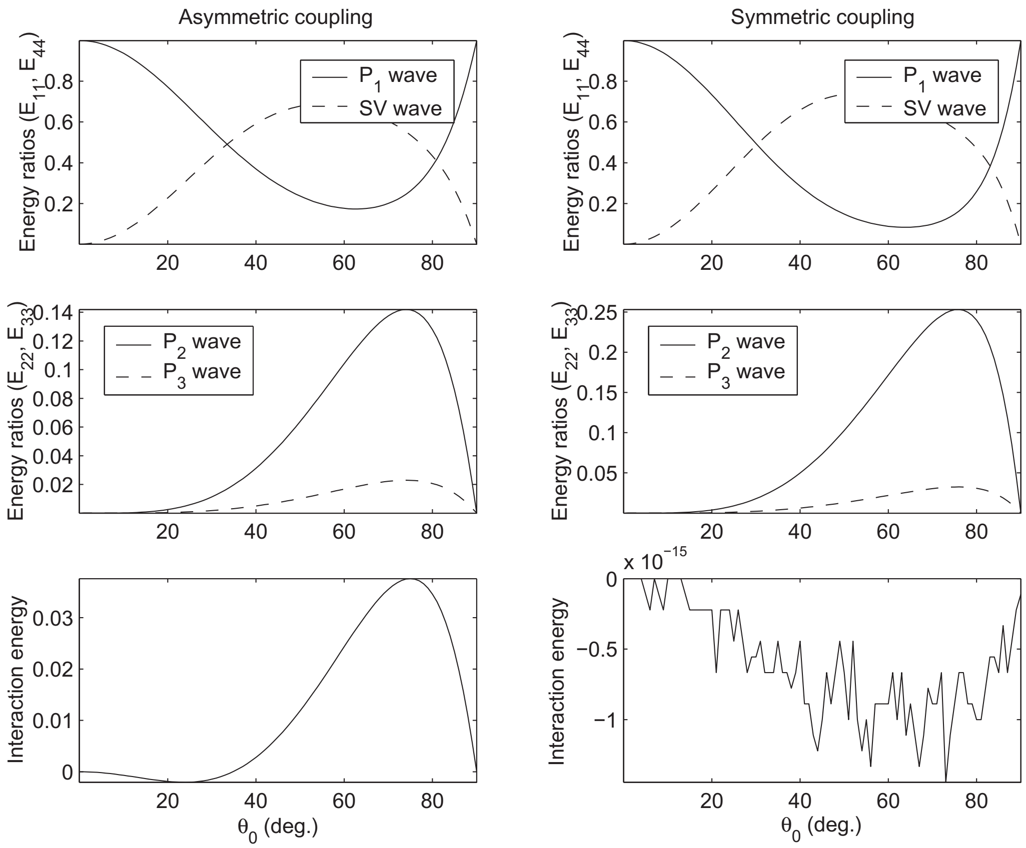

Fig. 1. Energy partition among the reflected waves from the incidence of P_1 wave at the pervious boundary of Berea sandstone; (E_{11}, E_{22}, E_{33}, E_{44}): energy fluxes of (P_1, P_2, P_3, SV) waves.

to derive certain restrictions and relations to be satisfied by the relevant phenomenological coefficients. But, the double porosity models are, generally, considered to be based on the mixture of solid and fluid phases (Wilson and Aifantis, 1984; Bai et al., 1993). In fact, the pore-pressure field may not be decoupled from the stress field in porous frame. On the contrary, a strong coupling exists between the solid deformation and the fluid flow in a porous composite. Hence, it becomes necessary to extend the Biot's concepts of simple poroelasticity to the double porosity models. Berryman and Wang (1995, 2000) made efforts for a rigorous extension of Biot's poroelasticity to include fractures/cracks. They derived the phenomenological equations and presented the method to determine the relevant coefficients for a generalized double-porosity/dual-permeability medium. Equations of motion were solved only for propagation of three compressional waves, which are showed to be dispersive and diffusive. The two slower modes find the lowest speeds and highest attenuations at low frequency. Based on the volume averaging technique, Pride and Berryman (2003a,b) derived the governing equations of fluid-saturated double porosity media. In addition, the fluid-transport mechanism was also investigated and a symmetric dual-permeability Darcy's law was established.

The present work considers the reflection of plane harmonic waves at the pervious (or impervious) plane boundary of a double porosity elastic solid. The conservation of incident energy is obtained numerically, through the unexpected presence of interaction energy. Changes in elastic properties, fracture/pore porosity, pore tortuosity and Skempton's coefficients could not help to eliminate this interaction energy. Only the symmetry in the coefficients of elastic coupling between the constituents ensures the complete distribution of incident energy among the four reflected waves. Else, to accommodate Skempton's coefficients (Skempton, 1954), the constitutive relations derived by Berryman and Wang (2000) need some modifications. The corrected constitutive relations are used to calculate the propagation velocities of the four waves in double porosity solid. Further, the reflection and refraction of an acoustic wave is also studied at the boundary of double porosity solid with liquid.

2. Basic equations

A double porosity medium consists of three constituents, i.e., solid matrix, pore-fluid and fracture-fluid. In this porous aggregate, volume fractions of the constituents are defined as

$$\delta_s = 1 - \phi, \quad \delta_1 = (1 - \epsilon)\phi, \quad \delta_2 = \epsilon\phi, \tag{1}$$

where, ϵ denotes the fraction of fractures in total porosity $\phi (= \delta_1 + \delta_2)$ of solid skeletal. The index 's' identifies the quantities corresponding to the solid phase in porous