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Effective properties of linear viscoelastic microcracked materials: Application of Maxwell homogenization scheme

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ABSTRACT

The present paper focuses on the effect of microcracks on the overall properties of viscoelastic materials. For this goal we extend Maxwell homogenization scheme to the case of viscoelasticity and derive explicit formulas for components of the anisotropic creep operator in dependence on scatter parameter characterizing orientation distribution of cracks. Microcracks can have any orientation distribution with randomly oriented and strictly parallel being the limiting cases. Viscoelastic behavior is described using fraction-exponential operators. The results are illustrated on example of microckracked polymethylmethacrylate (PMMA).

1. Introduction

In the present paper we focus on the effect of microcracks on the overall viscoelastic properties of hereditary materials. For this goal we extend Maxwell homogenization scheme to the case of viscoelasticity and derive explicit formulas for components of the anisotropic creep operator in dependence on scatter parameter characterizing orientation distribution of cracks. Viscoelastic behavior is described using fraction-exponential operators (see Rabotnov (1948, 1977)).

Generalization of the methods of micromechanics (developed originally for elastic heterogeneous materials) to viscoelastic composites have been proposed in a number of works in 1960s following remark of Eshelby (1957) that his results on elastic inclusion can be extended to linear viscoelastic materials. Hashin (1965) derived explicit relations for effective properties of viscoelastic composites with spherical and cylindrical inhomogeneities in terms of effective relaxation time and creep compliances. Schapery (1967) used Laplace–Carson transform to reduce the time-dependent homogenization problem into one of classic elasticity. To the best of our knowledge, Hashin (1970) first suggested to use elastic–viscoelastic correspondence principle to extend the classical homogenization schemes to the case of viscoelastic composites. This approach has been used for instance by Laws and McLaughlin (1978) who applied the self-consistent scheme to viscoelastic constituents. Wang and Weng (1992) and Brinson and Lin (1998) used it for generalization of Mori–Tanaka scheme; DeBotton and Tevet-Deree (2004) – for derivation of Hashin–Shtrikman bounds for viscoelastic composites. General bounds for the complex moduli of viscoelastic composites have been obtained by Gibiansky et al. (1999) using the Hashin–Shtrikman procedure and the translation method. Sanahuja (2013) compared different micromechanical scheme in the context of their predicting ability for aging linear viscoelastic composite with spherical inhomogeneities. Brenner and Suquet (2013), focusing on composites with Maxwellian constituents, obtained two asymptotic relations for the effective creep function of linear viscoelastic composites and discussed

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Main notations

$\ni^*_\alpha(\beta, t)$	fraction exponential operator
Greek letters	
α_{ij}	second rank crack density tensor
α	one of the parameters characterizing fraction-exponential operator $-1 < \alpha \leqslant 0$
β_{ijkl}	fourth rank crack density tensor
β	one of the parameters characterizing fraction-exponential operator ($\beta < 0$)
Γ	aspect ration of the effective inclusion in Maxwell's scheme
$\Delta\boldsymbol{\varepsilon}$	extra strain (per reference volume V) due to the presence of the inhomogeneity
$\boldsymbol{\varepsilon}^\infty$	the remotely applied strain
ζ	scatter parameter for cracks distribution
λ	one of the parameters characterizing fraction-exponential operator, $\beta < \lambda < 0$
μ_0	elastic shear modulus of the bulk material
μ_∞	shear modulus at $t \to \infty$
μ^*	shear operator of creep
$(\mu^*)^{-1}$	shear operator of relaxation
ν_0	elastic Poisson's ratio of the bulk material
ρ	scalar crack density
σ^∞_{kl}	remotely applied uniform stresses
φ	angle between normal to the crack surface and x_3-axis
Ω	effective inclusion in Maxwell's scheme
Latin letters	
$g_{1-6}(\zeta)$	functions describing dependence of the components of the crack density tensors on orientation distribution of cracks
$\boldsymbol{H}$	compliance contribution tensor
$\boldsymbol{H_{eff}}$	compliance contribution tensor of the effective inclusion in Maxwell's scheme
$\boldsymbol{H_i}$	compliance contribution tensor of ith inhomogeneity
$\tilde{h}_i$	coefficients representing tensor $\frac{\mu_0}{V^*}\sum_i V_i \boldsymbol{H_i}$ in tensor basis given in the Appendix A
K_0	bulk elastic modulus of the material
$K_{ijkl}(t)$	forth rank tensor creep kernel
$M_a(z)$	Mittag–Leffler function
$\boldsymbol{N}$	stiffness contribution tensor
$P_\zeta(\varphi)$	orientation distribution function for multiple cracks
$\boldsymbol{Q}$	one of two Hill's tensors
$\boldsymbol{Q}_\Omega$	is the Hill's tensor calculated for the shape of Ω
$\boldsymbol{S}^0$ and $\boldsymbol{S}^1$	are compliance tensors of the matrix and inhomogeneity
$\boldsymbol{S_{eff}}$	effective compliance calculated using Maxwell scheme (formula (2.19))
$\boldsymbol{S}^{lin}_{\boldsymbol{eff}}$	effective compliance calculated using Maxwell scheme linearized with respect to interaction parameter (formula (2.20))
$\boldsymbol{S}^{NIA}_{\boldsymbol{eff}}$	effective compliance calculated for non-interacting inhomogeneities
s_1	coefficients representing tensor $\boldsymbol{S_{eff}}$ in tensor basis
s^{lin}_3	coefficients representing tensor $\boldsymbol{S}^{lin}_{\boldsymbol{eff}}$ in tensor basis
$\boldsymbol{T}^{(i)}$	elements of standard tensor basis
V	reference volume
V_i	volume of ith inhomogeneity
V^*	volume of the effective inclusion in Maxwell's scheme

their physical meaning at short and long times. Their work improves previous one of Suquet (2012). Based on these relations, the authors proposed an approximate model for the creep function of a linear viscoelastic heterogeneous that approximates the retardation spectrum of the composite by a single discrete Dirac mass. The retardation time and its corresponding weight depend on the coupling between the elastic (resp. viscous) local compliances and the viscous (resp. elastic) stress field fluctuations.

Allen and Yoon (1998) formulated general principles of homogenization for heterogeneous linear thermoviscoelastic materials with stress–strain behavior of convolution type. The authors used elasticity–viscoelasticity correspondence principle and showed, in particular, that the macroscopically averaged relaxation moduli may be incorrect unless the time dependent nature of the strain localization tensor is taken into account. However, this error is negligible for most real materials.

In the context of microcracked viscoelastic materials, Zocher et al. (1997) were, probably the first ones who considered microcracked viscoelastic materials and compared analytical formulas, obtained with Laplace transform, with the finite elements calculations. Nguyen et al. (2011) built the effective moduli of a microcracked non-aging viscoelastic material that follows Burger model (combination of Maxwell and Kelvin sequences of dashpots and springs). Souza and Allen (2012) proposed a FEM procedure for determining the homogenized instantaneous (tangent) constitutive tensor of elastic materials containing growing cracks.

In all the mentioned problems, the approach to find analytical solution of the homogenization problem for a heterogeneous material with viscoelastic constituents is based on elasticity–viscoelasticity correspondence principle. The problem is formulated in the Fourier or Laplace domain, treated as the elastic one, and then, inverse transform gives the desired viscoelastic solution. The use of the Laplace transform has also been crucial in proving that short memory effects in the individual constituents give rise, after homogenization, to long memory effects in the composite (Sanchez-Hubert and Sanchez-Palencia, 1978; Suquet, 1986). The main challenge appearing in this approach is to obtain analytical formulas for inverse transform. It can be done only for some particular governing relations. Actually, governing relations of viscoelasticity (as all other relations of this kind) are of phenomenological nature – they are chosen from matching experimental data obtained from standard tests on creep and relaxation.