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A new method for smooth trajectory planning of robot manipulators

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Abstract

A new method for smooth trajectory planning of robot manipulators is described in this paper. In order to ensure that the resulting trajectory is smooth enough, an objective function containing a term proportional to the integral of the squared jerk (defined as the derivative of the acceleration) along the trajectory is considered. Moreover, a second term, proportional to the total execution time, is added to the expression of the objective function. In this way it is not necessary to define the total execution time before running the algorithm. Fifth-order B-splines are then used to compose the overall trajectory. With respect to other trajectory optimization techniques, the proposed method enables one to set kinematic constraints on the robot motion, expressed as upper bounds on the absolute values of velocity, acceleration and jerk. The algorithm has been tested in simulation yielding good results, which have also been compared with those provided by another important trajectory planning technique.

Keywords: Trajectory planning; Smoothness; Jerk; Robot manipulators; Optimization; B-splines

1. Introduction

A fundamental problem in robotics consists in trajectory planning, which may be defined in this way: find a temporal motion law along a given geometric path, such as certain requirements set on the trajectory properties are fulfilled. Trajectory planning is devoted to generate the reference inputs for the control system of the manipulator, so as to be able to execute the motion. The geometric path, the kinematic and dynamic constraints are the inputs of the trajectory planning algorithm, whereas the trajectory of the joints (or of the end effector), expressed as a time sequence of position, velocity and acceleration values, is the output.

The geometric path is usually defined in the operating space, i.e. with reference to the end effector of the robot, since both the task to perform and the obstacles to avoid can be more naturally described in this space.

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On the other side, trajectory planning is normally carried out in the joint space of the robot, after a kinematic inversion of the given geometric path. The joint trajectories are then obtained by means of interpolating functions which meet the imposed kinematic and dynamic constraints.

Planning a trajectory in the joint space rather than in the operating space has a major advantage, namely that the control system acts on the manipulator joints rather than on the end effector, so it would be easier to adjust the trajectory according to the design requirements if working in the joint space. Moreover, trajectory planning in the joint space would allow to avoid the problems arising with kinematic singularities and manipulator redundancy.

The main disadvantage of planning the trajectory in the joint space is that, given the planned trajectory in the joint space, the motion actually performed by the robot end effector is not easily foreseeable, due to the non-linearities introduced when transforming the trajectories of the joints into the trajectories of the end effector through direct kinematics.

Apart from the particular strategy adopted, the motion laws generated by the trajectory planner must fulfill the constraints set a priori on the maximum values of the generalized joint torques, and must be such that no mechanical resonance mode is excited. This can be achieved by forcing the trajectory planner to generate *smooth* trajectories, i.e. trajectories with good continuity features: in particular, it would be desirable to obtain trajectories with continuous joint accelerations, so that the absolute value of the *jerk* (i.e. of the derivative of the acceleration) keeps bounded. Limiting the jerk is very important, because high jerk values can wear out the robot structure, and heavily excite its resonance frequencies. Vibrations induced by non-smooth trajectories can damage the robot actuators, and introduce large errors while the robot is performing tasks such as trajectory tracking. Moreover, low-jerk trajectories can be executed more rapidly and accurately.

Almost every technique found in the scientific literature on the trajectory planning problem is based on the optimization of some parameter or some objective function. The most significant optimality criteria are:

(1) minimum execution time,
(2) minimum energy (or actuator effort),
(3) minimum jerk.

Besides the aforementioned approaches, some hybrid optimality criteria have also been proposed (e.g. time-energy optimal trajectory planning).

1.1. Minimum-time trajectory planning

Minimum-time algorithms were the first trajectory planning techniques proposed in the scientific literature because they were tightly linked to the need of increasing the productivity in the industrial sector. The first interesting methods of this kind [1,2] are developed in the position-velocity phase plane. The main idea is to use the curvilinear abscissa θ of the path as a parameter, in order to express the dynamic equation of the manipulator in a parametric form. An alternative approach is proposed in [3,4], where dynamic programming techniques are employed. However, the aforementioned techniques generate trajectories with discontinuous values of accelerations and joint torques, because the dynamic models used for trajectory computation assume the robot members as perfectly rigid and neglect the actuator dynamics. This leads to two undesired effects: first, the real actuators of the robot cannot generate discontinuous torques, which causes the joint motion to be always delayed with respect to the reference trajectory. The accuracy in trajectory following is then greatly reduced and the so-called chatter phenomenon may eventually occur, consisting in high frequency vibrations that can damage the manipulator structure. Second, the time-optimal control requires saturation of at least one robot actuator at any time instant, so the controller cannot correct the tracking errors arising from disturbances or modeling errors.

In order to overcome such problems, other approaches [5,6] impose some limits on the *actuator jerks*, defined as the variation rates of the joint torques. In this way, the generated trajectory will not be exactly time-optimal, albeit close to the optimality value; however, the generated trajectories can be effectively implemented and more advanced control strategies can be applied.