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Bridging strain gradient elasticity and plasticity toward general loading histories

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ABSTRACT

The Fleck–Hutchinson theory on strain gradient plasticity (SGP), proposed in Fleck and Hutchinson (1997), has been reformulated by adopting the strategy of decomposing the second order strain presented by Lam et al. (2003). This study attempts to build a SGP framework for general loading histories which has yet been well addressed. The main features of this study include: (1) The total number of the elastic characteristic length scales has been reduced from 5 to 3; (2) The anti-symmetric part of the rotational gradient has been found to have no influence on SGP; (3) The established SGP flow theory is characterized by its strict correspondence to the conventional J2 plasticity. This thermodynamically acceptable reformulation has been proven to satisfy the nonnegativity of plastic dissipation, which is still an outstanding issue in other SGP theories. It explicitly shows how elastic strain gradients and corresponding elastic characteristic length scales come into play in general elastic–plastic loading histories. Another feature of the present SGP formation is the exclusion of plastic strain-related boundary conditions which believably will facilitate SGP applications significantly.

1. Introduction

Important conceptions in the family of SGP theories are recalled via a brief review without seeking the wideness of literature coverage. Motivated by observed size effects in nonhomogeneous plastic deformations on the micron scale such as tests of microbend (e.g., Stölken and Evans, 1998), indentation (e.g., Nix and Gao, 1998), and wire torsion (e.g., Fleck et al., 1994), SGP theories were developed by many researchers with the emphasis on the role of strain gradients, based on the physics picture that both statistically stored dislocation (SSD) and geometrically necessary dislocation (GND) come into play. The fundamental SGP versions include Fleck et al. (1994), Fleck and Hutchinson (1997), Gao et al. (1999), Fleck and Hutchinson (2001), Gao and Huang (2001), Gundmundson (2004) and Fleck and Willis (2009). Despite of numerous successes that SGP has already achieved, some fundamental adjustments are in urgent needs, as discussed by for example, Hutchinson (2000), Evans and Hutchinson (2009) and Hutchinson (2012). Amongst existing issues, one major challenge is that SGP theories are not robustly workable in problems with general loading histories, for example loading–unloading cycles. Other issues announced by Hutchinson (2012), such as unreasonable discontinuous changes in higher order stresses upon certain infinitesimal load changes and failure in guaranteeing nonnegative dissipations, are believed to arise also due to lack of robust SGP theory for complex loading histories.

The omission of the strain gradient elasticity effect, as done in the SGP theories like Fleck and Hutchinson (2001)

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and Gundmundson (2004), directly leads to discontinuous changes in *total* strain gradients during complex loading histories, and therefore causes the fundamental issues summarized by Hutchinson (2012). Consideration of elastic strain gradient requires a good understanding of strain gradient elasticity (SGE) theories, which have been described in details by Fleck and Hutchinson (1997) on formulations of kinematics, kinetics, as well as equilibrium and boundary conditions. In order to ease the complexity in dealing with 5 elastic material characteristic length scales appearing in the expression of elastic second order strain energy density, Lam et al. (2003) proposed a new decomposition strategy of second order strain and, hence, reduced the total number of elastic length scales to 3, based on the conclusion that the anti-symmetric part of the rotational gradient should not enter the expression of the second order strain energy density. Notably, the strain gradient decomposition proposed by Lam et al. (2003) was an important theoretical advancement even though the elastic length scales were controversially argued to be on the order of micron. In the present study, an attempt is made to combine the above-mentioned theories in a compatible manner, to formulate a SGE-SGP framework, which is expected to possess the capacity for cases with general loading histories.

Generalization of the conventional flow theory will be made to establish the present SGP theory. Many significant advances in the history of plasticity theories were accomplished by generalization or extension of certain conclusions for some simpler cases. For example, the three-dimensional formulations of flow theory and Drucker's postulate were obtained by generalizing the one-dimensional counterparts which are based on uniaxial tension tests. In handling SGP, a generalization of treating variables of the same dimension in an analogous manner has been commonly adopted, for example the plastic strain gradient multiplied by the plastic characteristic length scale was taken as the plastic strain, and the higher order stress divided by the material length scale was treated in the same manner as the lower order stress, i.e., Cauchy stress. The above-mentioned generalization will be also adopted for the present study. Furthermore, since there are two sets of length scales, i.e. the elastic one and the plastic one, an emphasis will be made about their proper usages.

The paper is organized as follows. A summary of the SGE theory is provided in Section 2. Subsequently, in Section 3 the conventional J2 flow theory is revisited and used as a template for generalization of the conventional plasticity in the SGE-SGP settings including development of a new flow theory and satisfaction of Drucker's postulate with the consideration of elastic–plastic second order strain decomposition. The wire torsion problem is studied as an example to show the ability of the present theory in Section 4. This paper ends with conclusions and discussions in Section 5.

2. SGE framework

In this section, the SGE theory presented by Lam et al. (2003) is reformulated to link with the SGP theory. For a general linear isotropic elastic solid, the general SGE formulation is firstly introduced, which can be reduced to the couple stress version by excluding all other strain gradient components except the rotational gradient. To achieve a compatible SGE-SGP framework during elastic–plastic deformation processes that will be studied in following sections, the necessary generalizations of the theory in Lam et al. (2003) will be made, providing a detailed introduction of the platform from which generalization of the conventional plasticity will be carried out.

The generalized strain variables are the symmetric strain tensor ϵ_{ij} and the second order gradient of displacement η_{ijk}, which are expressed, respectively, as

$$\epsilon_{ij} = \epsilon_{ji} = \frac{1}{2}(\partial_i u_j + \partial_j u_i), \quad \eta_{ijk} = \eta_{jik} = u_{k,ij} = \partial_{ij} u_k, \tag{2.1}$$

where u_i is the ith displacement component and ∂_i is the forward gradient operator.

The major difference between the conventional elasticity and SGE is that the change in elastic strain energy density, $\dot{W}$, depends on the changes in both ϵ_{ij} and η_{ijk} due to an arbitrary infinitesimal variation of displacement $\boldsymbol{u}$. In the energetic calculation, in order to maintain consistency with the following SGP formulations, the superscript "e" which denotes "elasticity", is added onto strains and second order strains as well as the strain energy, which makes no change in the present section because there are $\boldsymbol{\epsilon}^e = \boldsymbol{\epsilon}$, $\boldsymbol{\eta}^e = \boldsymbol{\eta}$ and therefore $W^e = W$ for pure linear elastic problems. The change in energy is written as,

$$\dot{W}^e = \sigma_{ij}\dot{\epsilon}^e_{ij} + \tau_{ijk}\dot{\eta}^e_{ijk}, \tag{2.2}$$

where the symmetric Cauchy stress σ_{ij} $(=\sigma_{ji})$ and the higher-order stress τ_{ijk} $(=\tau_{jik})$ are the work conjugates of respectively the elastic strain and second order strain, $\epsilon^e_{ij} = \int \dot{\epsilon}^e_{ij} dt$ and $\eta^e_{ijk} = \int \dot{\eta}^e_{ijk} dt$. The higher order stress tensor is composed of both couple stresses and double stresses. The work statement Eq. (2.2) gives the following elastic constitutive relations,

$$\sigma_{ij} = \frac{\partial W^e}{\partial \epsilon^e_{ij}}, \quad \tau_{ijk} = \frac{\partial W^e}{\partial \eta^e_{ijk}}. \tag{2.3}$$

It is worth noting that in the following τ_{ijk} will not be given directly but can be constructed by putting together the work conjugates of particular compositions of the second order strain $\boldsymbol{\eta}$. In the following we adopt the SGE formulation by Lam et al. (2003). For comparison purpose, the formulation used by Fleck and Hutchinson (1997) is presented in Appendix A.

Firstly, by decomposing the second order strain $\boldsymbol{\eta}$ into symmetric and anti-symmetric parts via the strategy proposed by Fleck and Hutchinson (1997), we obtain,

$$\begin{aligned} &\boldsymbol{\eta} = \boldsymbol{\eta}^S + \boldsymbol{\eta}^A, \eta^S_{ijk} = \frac{1}{3}\left(\eta_{ijk} + \eta_{jki} + \eta_{kij}\right), \\ &\eta^A_{ijk} = \frac{2}{3}\left(e_{ikp}\chi_{pj} + e_{jkp}\chi_{pi}\right). \end{aligned} \tag{2.4}$$

where χ is the rotational gradient.

Subsequently, new independent second order strain metrics are obtained by splitting the symmetric second order strain η^S_{ijk} into a trace part $\eta^{(0)}_{ijk}$ and a traceless part $\eta^{(1)}_{ijk}$, as follow: