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A creep model with damage based on internal variable theory and its fundamental properties

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ABSTRACT

The creep damage is discussed within Rice irreversible internal state variable (ISV) thermodynamic theory. An ISV small-strain unified creep model with damage is derived by giving the complementary energy density function and kinetic equations of ISVs. The proposed model can describe viscoelasticity and, preferably, three phases of creep deformation. Creep strain results from internal structural adjustment, and different creep stages accompany different thermodynamic properties in terms of flow potential function and energy dissipation rate. During the viscoelastic process, the thermodynamic state of the material system tends to equilibrate spontaneously. The thermodynamic state of the material system without damage tends to equilibrate or achieve steady state after loading. Kinetic equations of ISVs can be derived by one single flow potential function, and the energy dissipation rate decreases monotonically over time. In the entire creep damage process, multiple potentials are needed to characterise evolution of ISVs, rotational fluxes are presented in affinity space, and the thermodynamic state of material system tends to depart from the steady or equilibrium state. The energy dissipation rate can be a measure of the distance between the current thermodynamic state and the equilibrium state. The time derivative of the rate can characterise the development trend of the material, and the integral value in the domain may be regarded as indices to evaluate the long-term stability of the structure.

1. Introduction

The creep property of rocky mass is important to the long-term stability of geotechnical engineering (Yang and Jiang, 2010). The time-dependent mechanical behaviour of the rocky mass in a complex geological environment can be described by the creep model, which is the key for stability analysis and safety evaluation. The creep model based on the irreversible internal state variable (ISV) thermodynamic theory is thermodynamically consistent and could characterise the intrinsic energy dissipation process and physical changes in the microstructure of material (Lubliner, 1972; Park et al., 1996; Zhu and Sun, 2013).

At present, most time-dependent models are either viscoelasticity or viscoplasticity. A unified creep model in which both viscoelasticity and viscoplasticity can be considered is desirable (Henry and Haslach, 2002). Because energy dissipation would happen in both viscoelastic and viscoplastic deformation processes, the viscoelasticity and viscoplasticity can be considered in the same irreversible thermodynamic framework. For example, a viscoelastic equation depends on Newton time, but the viscoplastic equation depends on an “intrinsic time” for the model from Valanis (1971). Schapery (1997) introduced three sets of ISVs for constructing nonlinear viscoelastic and viscoplastic constitutive equations. Zhu and Sun (2013) proposed a unified model for asphalt concrete based on irreversible thermodynamics.

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In practice, creep deformation of materials is accompanied by a damage process or even a healing process (Abu Al-Rub et al., 2010; Voyiadjis et al., 2011, 2012; Sun et al., 2013), and investigation of a creep model with damage has received more and more attention in the literature (Cristescu, 1986; Chan et al., 1992, 1997; Betten et al., 1998; Voyiadjis and Zolochevsky, 1998; Challamel et al., 2005). Two ways can traditionally take the damage effect into creep processes. One of these ways is that a damage variable with evolution law, which could be scalars, vectors, second-order, or fourth-order tensors, are introduced to characterise the degradation effect of material parameters (Richard et al., 2010; Zhou et al., 2013; Wang et al., 2013). The other way is that a term resulting from damage can be added to the original creep model (Chan et al., 1997; Yang et al., 2002; Chen et al., 2006). In internal variable theory, damage variables of the creep model are regarded as ISVs, and their evolution laws could be derived through thermodynamic potentials and a maximum dissipation principle similar to other ISVs. In this case, multi-potential functions are needed. For example, Hansen and Schreyer (1994) used the concept of effective stress to develop a unified framework of coupled elastoplastic and damage theories, in which the hardening variables and damage variables were derived from plasticity and damage potentials, respectively. Chaboche (1997) introduced three thermodynamic potentials to develop coupled viscoplastic and damage formulations for metal and polymers. Voyiadjis and Zolochevsky (2000) employed three independent dissipation potentials to construct a constitutive model of creep damage in polycrystalline metals and alloys with different behaviour in tension and compression.

The viscoplastic or plastic strain is regarded as an ISV that is conjugated to stress in most ISV creep models at present. Its evolution law could be determined directly by the dissipation function and its orthogonality condition. However, within the Rice irreversible ISV thermodynamic framework, a set of scalar ISVs is used to present the internal physical changes of the microstructure of material. Plastic strain is not an ISV but only a variable result from the inner structural change of material (Rice, 1971). Rice's thermodynamic theory can be used to connect the change of the microstructure to the relative deformation of material and obtain more and more attention in the field of material study. Li (2013) discussed complex plastic deformation of geotechnical materials based on Rice's theory. Sherburn et al. (2011) used Rice flow potential functions to determine the evolution of ISVs in their proposed inelastic constitutive equation. Yang et al. (2005a) discussed some fundamental issues in damage evolution laws for microcracked solids within the framework of normality structures by Rice.

In this paper, the creep damage is discussed within Rice's ISV thermodynamic framework. Three sets of macroscale ISVs are introduced to describe the change of internal structural adjustment of material in the creep process. A creep model with damage is developed by giving the specific complementary energy and evolution of internal variables. Next, the flow potential and evolution of the intrinsic energy dissipation rate are discussed in different creep phases, which is of key significance for establishing an evaluation index for the long-term stability of the geotechnical project. In this work, we assume that the material system is isotropic and subjected to small deformation. In Section 3, the performance of the proposed model is surveyed by experimental data that show that the creep strain of the test material, even in the tertiary creep stage, is smaller than 10%; thus, the small strain assumption holds for the case of creep damage investigated in this study.

2. Rice's ISV thermodynamic theory

2.1. Thermodynamics of constrained equilibrium states

The Rice irreversible ISV thermodynamics based on a constrained equilibrium state assumes that the state of solid material at any given time can be described fully by the stress $\boldsymbol{\sigma}$ or strain $\boldsymbol{\varepsilon}$, the temperature θ, and a set of local scalar internal state variables $\xi(\xi_1, \xi_2, \ldots, \xi_n)$ that present the physical changes of microstructures within the material (Rice, 1971). Stress $\boldsymbol{\sigma}$ or strain $\boldsymbol{\varepsilon}$, temperature θ and ISVs ξ are referred to as thermodynamic state variables. The specific free energy ϕ and specific complementary energy ψ are the main thermodynamic potential functions and satisfy the Legendre transform (Rice, 1971; Yang et al., 2005b),

$$\phi(\boldsymbol{\varepsilon}, \theta, \xi) + \psi(\boldsymbol{\sigma}, \theta, \xi) = \boldsymbol{\varepsilon} : \boldsymbol{\sigma} \tag{1}$$

$$\boldsymbol{\sigma}(\boldsymbol{\varepsilon}, \theta, \xi) = \frac{\partial \phi(\boldsymbol{\varepsilon}, \theta, \xi)}{\partial \boldsymbol{\varepsilon}} \tag{2}$$

Considering the neighbouring constrained equilibrium states, the equation corresponding to different sets of state variables can be written like thermostatics in the following equation (Rice, 1970),

$$\delta\psi = \boldsymbol{\varepsilon} : \delta\boldsymbol{\sigma} + \theta\delta\eta + \frac{1}{V} f_\alpha \delta\xi_\alpha \tag{3}$$

where η is the total specific entropy; V is material volume; $\boldsymbol{f}(f_1, f_2, \ldots, f_n)$ denotes the thermodynamic forces that are conjugated to the local ISV ξ. Einstein's summation convention is adopted for repeated indices unless otherwise indicated as Eq. (44) in this work. The following relationships can be derived through the above-mentioned equations.

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}(\boldsymbol{\sigma}, \theta, \xi) = \frac{\partial \psi(\boldsymbol{\sigma}, \theta, \xi)}{\partial \boldsymbol{\sigma}} \tag{4}$$

$$\eta = \frac{\partial \psi(\boldsymbol{\sigma}, \theta, \xi)}{\partial \theta} \tag{5}$$

$$f_\alpha = V \frac{\partial \psi}{\partial \xi_\alpha} = -V \frac{\partial \phi}{\partial \xi_\alpha} \tag{6}$$

Thermodynamic forces $\boldsymbol{f}$ are known to conjugate to the local ISV ξ, stress $\boldsymbol{\sigma}$ to strain $\boldsymbol{\varepsilon}$ and temperature θ to specific entropy η. The incremental form of Eq. (3) can be extended as a rate equation to represent dynamic processes (Yang et al., 2009).

$$\dot{\psi} = \boldsymbol{\varepsilon} : \dot{\boldsymbol{\sigma}} + \eta\dot{\theta} + \frac{1}{V} f_\alpha \dot{\xi}_\alpha \tag{7}$$