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A model for predicting uniaxial compression behavior of fused fibrous networks

Vijay Kumar, Amit Rawal *

Department of Textile Technology, Indian Institute of Technology Delhi, Hauz Khas, New Delhi, India

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ABSTRACT

Fused fibrous networks are increasingly being used for emerging industrial applications ranging from thermal/sound insulation, fluid filtration/separation, and energy conversion to tissue scaffolds. Majority of these applications need a deeper understanding of fused fibrous networks under compression loading. In this research work, a compression model of fused fibrous networks has been proposed by defining two distinct regions displaying the bending of free fiber segments between the fiber-to-fiber contacts followed by the transverse compression of fiber contacts through classical Hertzian contact mechanics approach. The mechanistic models developed in this study, have clearly elucidated the main fiber and structural parameters that control the compression behavior of fused fibrous networks. A comparison has also been made between the theoretical and experimental pressure–strain curves of randomly and preferentially aligned fused fibrous networks.

1. Introduction

Fiber networks form constituent materials for paper, felts, carbon nanotubes, electrospun mats to interconnected networks of filamentous proteins (Atwater et al., 2013; Broedersz et al., 2011; Choong et al., 2013; Deng and Dodson, 1994; Ridruejo, 2011). The mechanics of these fibrous networks essentially depend upon elastic properties of constituent fibers, alignment of fibers in the network and connectivity of the network. On applying the load to the fibrous network, the constituent fibers act as load bearing elements and the connectivity of the network assists in transferring the load to the fiber segments. In general, the mechanical properties of network of fibers can be significantly improved by fusing the contacts between the fibers (Choong et al., 2013; Huang et al., 2013; Mannarino and Rutledge, 2012). Accordingly, the fused fibrous networks are expected to have an improvement in filtration/separation properties, thermal/sound insulation, charge collection, energy conversion and many other characteristics. For example, the electrospun mat having network of fused fibers can sustain high pressure operations such as reverse osmosis corresponding to the mat having frictional contacts between the fibers (Choong et al., 2013). In addition, such high pressure operations would cause uniaxial compression loading to the fused fibrous network. Thus, an understanding of the compression behavior of fused fibrous network is critical and of paramount importance. Moreover, the degree of compression loading applied to the network of fibers can significantly alter their other characteristics such as electrical conductivity, strain sensing, energy absorbing, etc. (Camilli et al., 2013; Wu et al., 2012).

Numerous investigations have focused on understanding the compression behavior of fibrous networks based on empirical and micromechanical models (Cox, 1952; Carnaby and Pan, 1989; Komori and Itoh, 1991a, 1991b;

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* Corresponding author. Tel./fax: +91 11 26591472.
E-mail address: amitrawal77@hotmail.com (A. Rawal).

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Komori et al., 1992; Lee and Carnaby, 1992; Lee and Lee, 1985; Toll, 1998; van Wyk, 1946). Majority of these compression models were formulated by considering fiber bending as the main deformation mechanism (Carnaby and Pan, 1989; Komori and Itoh, 1991a; Lee and Carnaby, 1992; Lee and Lee, 1985; van Wyk, 1946). Fiber bending has played a key role in the meso- and macro-scopic mechanics for a wide range of fibrous networks under moderate levels of compression loading (Broedersz et al., 2011). To date, the authors are not aware of any studies that predict the compression behavior of fiber networks specifically under a wide range of stresses. Although, a number of researchers attempted to predict the compression behavior of *general fibrous assemblies* including nonwovens based on stochastic and stereological approaches (Carnaby and Pan, 1989; Komori and Itoh, 1991a, 1991b; Komori et al., 1992; Lee and Carnaby, 1992; Lee and Lee, 1985; Rawal, 2009; Wu and Dzenis, 2005; van Wyk, 1946). However, in this research work, a first attempt has been made to predict the compressibility of fused fibrous networks under a wide range of stresses by combining stochastic, stereological and Hertzian contact mechanics approaches. A comparison has also been made between theoretical and experimental results of compression behavior of preferentially and randomly aligned fused fibrous networks. In the past, some of the models developed for fibrous networks are defined by two aligned fibers at the micro- or nano- scales (Wu and Dzenis, 2007; Wu et al., 2012). In reality, the fibers in the network exhibit a range of alignments and the distance between the fiber-to-fiber contacts is variable in nature. Thus, the structural parameters of fibrous networks need to be expressed in terms of structural variables by including their all possible fiber alignments leading to use of both stereological and stochastic approaches.

2. Theoretical analysis

In general, the compression behavior of fused fibrous networks can be defined in terms of two distinct regions, as illustrated in Fig. 1. Region I is defined by a non-linear curve that indicates *bending of free fiber segments* between fiber-to-fiber contacts under low and moderate levels of compression loading. Here, the compression strain rapidly increases with an increase in the magnitude of pressure as there is a significant enhancement in the number of fiber-to-fiber contacts due to higher deflection of mean free fiber element between the contacts. Further increase in the compressive stresses has reduced the mean free fiber element existing between the contacts resulting in lower deflection and the fibers deform spuriously to develop 'pseudo asymptote' indicating the onset of region II in the pressure–strain curve. In region II, transverse compression has been imposed by the fibers against each other without any significant changes in their spatial orientation. Hence, the fiber segments would simply press against each other over a defined region of contact area by means of *Hertzian contact forces*. This region of contact area between the fibers and its pressure distribution would strongly depend upon the alignment of fibers, inflicted from region I. The transition from region I to region II is dictated by computing the ratio of fiber deformations in respective regions. Accordingly, the compression model is tuned to region I when the deformation computed from bending of free fiber elements is greater than that of deformation predicted from Hertzian contact forces, i.e. $(\bar{\delta}_j)_{Hertz} < (\bar{\delta}_j)_{bend}$. Similarly, the compression model is extended to region II when the fiber deformations predicted from region II exceeds from the corresponding deformation obtained from region I, i.e. $(\bar{\delta}_j)_{Hertz} > (\bar{\delta}_j)_{bend}$, also illustrated in Fig. 1. It is worth mentioning that randomly aligned network of fibers exhibit uniform mean free fiber length and accordingly, uniform deflection of mean free length takes place that results in an earlier onset of 'pseudo asymptote' in the pressure–strain curve (Rawal, 2009).

A network of fibers essentially consists of fibers aligned randomly or preferentially (majority of the fibers are aligned in one direction) and accordingly, these fiber segments are irregularly spaced and distributed in a stochastic manner. Consider an element of fiber having a mean distance between the centers of two adjacent fiber-to-fiber contacts ($\bar{b}$) whose direction with respect to the spherical coordinate system is defined by polar (θ) and azimuthal (φ) angles, as

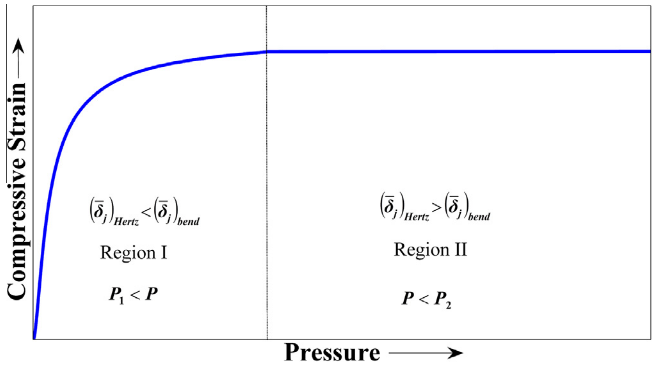

Fig. 1. A typical pressure–strain curve of fused fibrous network depicting two distinct regions I and II. Here $(\bar{\delta}_j)_{bend}$ and $(\bar{\delta}_j)_{Hertz}$ are the deformations occurred due to bending of free fiber segments and that of fiber-to-fiber contacts by means of Hertzian contact forces in the direction of the compressive stresses (j), respectively.