Mechanics of Materials 77 (2014) 28–42

Contents lists available at ScienceDirect

Mechanics of Materials

journal homepage: www.elsevier.com/locate/mechmat

Modeling of the contact between a rigid indenter and a heterogeneous viscoelastic material

Koffi Espoir Koumi [a,b], Daniel Nelias [a,*], Thibaut Chaise [a], Arnaud Duval [a]

[a] *Université de Lyon, INSA-Lyon, LaMCoS UMR CNRS 5259, F69621 Villeurbanne, France*
[b] *SNECMA, Centre de Villaroche, 77550 Moissy Cramayel, France*

ARTICLE INFO

Article history:
Received 10 April 2014
Received in revised form 30 June 2014
Available online 10 July 2014

Keywords:
Viscoelasticity
Inhomogeneity
Anisotropy
Eigenstrain
Eshelby's equivalent inclusion method (EIM)
Contact Mechanics

ABSTRACT

In this paper, the contact problem between a rigid indenter and a viscoelastic half space containing either isotropic or anisotropic elastic inhomogeneities is solved. The model presented here is 3D and based on semi-analytical methods. To take into account the viscoelastic properties of the matrix, contact and subsurface problem equations are discretized in the spatial and temporal dimensions. A conjugate gradient method and the fast Fourier transform are used to solve the normal problem, contact pressure, subsurface problem and real contact area simultaneously. The Eshelby's formalism is applied at each step of the temporal discretization to account for the effect of the inhomogeneity on pressure distribution and subsurface stresses. This method can be seen as an enrichment technique where the enrichment fields from heterogeneous solutions are superimposed to the homogeneous viscoelastic problem solution. Note that both problems are fully coupled. The model is validated by comparison with a Finite Element Model. A parametric analysis of the effect of elastic properties and geometrical features of the inhomogeneity is proposed. The model allows to obtain the contact pressure distribution disturbed by the presence of inhomogeneities as well as subsurface and matrix/inhomogeneity interface stresses at every step of the temporal discretization.

1. Introduction

Composite materials are more and more used in several domains of the engineering: aeronautics, automobiles ...Some of these composite materials are made of by viscoelastic matrix and generally reinforced with anisotropic elastic fibers. Considering the complexity of these materials, it is difficult to develop a mathematical model allowing to analyze or predict their behavior within the framework of contact mechanics. Several authors have shown interest to the problem of contact between homogeneous viscoelastic materials. Lee and Radok (1960) obtained the solution of contact pressure and area for the spherical indentation of a linear viscoelastic material. This solution is valid only for monotonical increase of the contact area and leads to a negative pressure when the contact area decreases. Since then, several approaches were proposed in the literature to overcome this issue. Hunter (1960) and Graham (1967) introduced methods for viscoelastic indentation test which are able to handle the case when the contact radius possesses a single maximum. Ting (1966, 1968) dealt with the indentation problem in which the time dependent contact area is an arbitrary function of time, and the indenter has an axisymmetric profile. Most of the solutions presented above are based on complex analytical formalisms, and limited to particular geometries (cone, sphere) and to the ideal viscoelastic material with only one relaxation time. Several other authors were interested in the indentation problem, rolling, sliding and

* Corresponding author.
E-mail address: daniel.nelias@insa-lyon.fr (D. Nelias).

http://dx.doi.org/10.1016/j.mechmat.2014.07.001
0167-6636/

Nomenclature

Letters

a^*	contact radius
a_1, a_2, a_3	semi-axes of an ellipsoidal inhomogeneity
B^*_{ijkl}	influence coefficients that relating the stress σ_{ij} at point (x_1^3, x_2, x_3) to the constant eigenstrain at the point (x_1^k, x_2^k, x_3^k)
C^M_{ijkl}, C^I_{ijkl}	elastic constants of the matrix and the inhomogeneity
E^I	Young's modulus of the inhomogeneity
h	distance between the two surfaces of the contacting bodies
$H(t)$	the Heaviside step function
I_{ijkl}	the fourth-order identity tensor
$J(t)$	viscoelastic creep function
K^n	coefficients in the normal displacement at the contact surface due to the contact pressure
L_1, L_2, L_3	lengths of the three sides of the matrix in Finite Element Model
M_{ij}	influence coefficients relating the stress σ_{ij} at the point (x_1, x_2, x_3) to the normal traction σ^n within a discretized area centered at $(x_1^k, x_2^k, 0)$
n_1, n_2, n_3	grid sizes in the half-space along the Cartesian directions x_1, x_2, x_3, respectively
P	normal applied load
P_0	maximum Hertzian pressure
p	contact pressure distribution
D	indenter diameter
$R(t)$	viscoelastic relaxation function
S_{ijkl}	components of the Eshelby's tensor
u_i^0	displacements corresponding to the infinite applied strain ε^0_{ij}
u_i	disturbed contribution of the displacements
W	applied exterior load
dx_3	depth of the inclusion from the surface of the matrix in EF model
$x^I = (x_1^I, x_2^I, x_3^I)$	Cartesian coordinates of the inclusion center

Greek letters

ε^0_{ij}	infinite applied strain
ε_{ij}	strain due to eigenstrains
ε^*_{ij}	eigenstrain due to the presence of inhomogeneities
σ^0_{ij}	stress corresponding to the infinite applied strain ε^0_{ij}
σ_{ij}	disturbed contribution of the stresses
ϕ, Ψ	harmonic and biharmonic potentials of mass density ε^*_{ij}
δ_{ij}	Kronecker symbol
σ^n	normal pressure due to the summation of both symmetric inclusions
$\Delta x_1, \Delta x_2$	half size of the discretized surface area
ν^M, ν^I	Poisson's ratio of the matrix M and the inclusion I
γ	the ratio of the inhomogeneity Young's modulus to the matrix
η	the dashpot viscosity
τ	the relaxation time
θ	the tilted angle of the inhomogeneity in the x_1Ox_3 plan

Acronyms and fast Fourier transforms

2D-FFT	two-dimensional fast Fourier transform
3D-FFT	three-dimensional fast Fourier transform
FFT^{-1}	inverse FFT operation
$\widehat{B}_{ijkl}$	frequency response of coefficients B_{ijkl} in the frequency domain
$\widehat{M}_{ij}$	frequency response of coefficients M_{ij} in the frequency domain

rolling friction for viscoelastic materials. This is the case of Argatov (2012) who gave the analytical solution of the rebound indentation problem for an isotropic linear viscoelastic layer loaded with a spherical punch. His model is only valid for the indentation phase with a monotonic loading. A contact between an axisymmetric indenter and a viscoelastic half-space is presented by Greenwood (2010). Considering the sliding contact or rolling friction in viscoelastic contact problem; Hunter (1961) and later, Goriacheva (1973) proposed a solution for the rolling contact between a rigid indenter and a viscoelastic half-space. Both approaches have in common to limit themselves to particular shape (cylindrical) of the indenter and to an ideal viscoelastic material. To get rid of the indenter shape limitation, Vollebregt (2011) proposed a model based on boundary element method. His model can only account for viscoelastic material with one single relaxation time. Recently, Carbone and Putignano (2013) introduced a novel methodology to investigate steady-state viscoelastic sliding or rolling contact based on boundary element method. Their approach is able to deal with a large spectra of relaxation times for the viscoelastic materials. Other authors (Nasdala et al., 1998; Le and Rahler, 1994; Nackenhorst, 2004; Padovan et al., 1992, 1984) solved the sliding/rolling friction problem of viscoelastic contact using the Finite Element Method. This method is able to account for the real viscoelastic materials and any geometry of contacting bodies. However using the Finite Element Method, the accuracy of the contact solution in terms of pressure and subsurface stresses is often insufficient. A robust semi-analytic method for contact between a rigid indenter and a viscoelastic half-space has first been introduced by Chen et al. (2011). This semi-analytical approach can account: for a wide spectra of relaxation times for linearly viscoelastic materials, an arbitrary loading profile (with possibility of decreasing the contact area); it can also simulate contact with a rough surface by incorporating the asperity heights into initial surface gap. The model presented in this paper is based on the semi-analytical method introduced by Chen et al. (2011) for viscoelastic