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Finite element analysis of frictionless contact between a sinusoidal asperity and a rigid plane: Elastic and initially plastic deformations

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ABSTRACT

A series of finite element simulations of frictionless contact deformations between a sinusoidal asperity and a rigid flat are presented. Explicit expressions of critical variables at plastic inception including interference, contact radius, depth of first yielding, and pressures are obtained from curve fitting of simulation results as a function of material and geometrical parameters. It is found Hertz solution is not applicable to the critical contact variables at plastic inception for sinusoidal contact, although contact responses of initially plastic deformation follow the same trend as that of purely elastic deformation. The contact pressure at incipient plasticity, which is defined as yield strength, is dependent on Poisson's ratio, yield stress, and geometrical parameters, but independent of elastic modulus. It is not yield stress, but yield strength that correlates with indentation hardness. The results yield the insight into the specification of material properties to realize elastic contact. A larger ratio of yield stress to elastic modulus is beneficial to sustain a larger load before plastic deformation.

1. Introduction

An increasing number of studies have been conducted with the objective of achieving a fundamental understanding of the contact behavior in both elastic and plastic regimes. Much attention has been paid on single asperity contacts (Chang et al., 1987; Zhao et al., 2000; Mihailidis et al., 2001), since highly localized deformation can be used as a unit event, which can be incorporated into statistical models of rough surface consisting of multiple asperities (Greenwood and Williamson, 1966; Whitehouse and Archard, 1970; Nayak, 1971, 1973; Kucharski et al., 1994; Cohen et al., 2009), and is of critical importance in providing much insight into many problems (Cha et al., 2004), such as sealing, friction, thermal and electrical conductance. Most of the researchers have concentrated on the contact between a deformable sphere and a rigid flat (Etsion et al., 2005b; Ovcharenko et al., 2008; Chatterjee and Sahoo, 2013), since a wide range of applications consist of interactions resembling spherical contacts (Eriten et al., 2012). Many studies on contact involving sinusoidal asperities have been motivated as well (Yastrebov et al., 2011), since a rough surface can be idealized with a sinusoidal profile (Gao et al., 2006; Sun et al., 2012). Westergaard (1939) and Dundurs et al. (1973) solved the contact pressure and area for the elastic contact of a one-dimensional sinusoidal surface with a flat. Johnson et al. (1985) concerned the elastic contact of a two-dimensional sinusoidal surface with a flat. Dundurs et al. (1973) studied contact between two elastic semi-infinite bodies with frictionless contact condition and periodic sinusoidal surface

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profiles. For mathematical tractability of the problem, infinitesimal plain strain deformation was assumed (Gao et al., 2006), and the boundary conditions were not imposed on the slightly curved surfaces (Dundurs et al., 1973).

Elastic–plastic contact (Liu and Yang, 2013a; Liu and Yang, 2013b) can be investigated more accurately by finite element approach (Kucharski et al., 1994; Etsion et al., 2005b; Eid et al., 2011) owing to the inherent shortcomings of theoretical approaches (e.g. small deformation Hill et al., 1989; Hill, 1992; Storåker et al., 1997 and discontinuous pressure gradient at contact edges Westergaard, 1939). Difference between finite element models and theoretical models can be remarkable (Kucharski et al., 1994; Buczkowski and Kleiber, 2006). Mesarovic and Fleck (1999) used finite element method to study the normal contact of a rigid sphere and an elastic–plastic half-space, and found the restricted validity of the rigid-strain-hardening similarity solution. Song and Komvopoulos (2013) examined the deformation of an elastic–plastic half-space by a rigid spherical indenter. Their finite element results showed semi-logarithmic dependence of mean contact pressure on indentation depth (Ye and Komvopoulos, 2003; Kogut and Komvopoulos, 2004) held only in the deformation regime of fully confined plastic zone, and the post-yield contact response consisted of four deformation regimes (linear elastic–plastic, nonlinear elastic–plastic, transient fully plastic, and steady-state fully plastic), which is contrary to classical contact mechanics showing only two deformation regimes (elastic–plastic, and fully plastic deformation) after yielding (Johnson, 1985).

Understanding the initial stage of contact that occurs between asperities of micro- or nano-scope dimensions is of fundamentally importance. Enormous research interest has been focused on determination of the onset of plastic deformation for a given material under a given loading conditions (Minor et al., 2006). The load required for yielding and the location of the onset of plasticity are critical in the robustness of conventional as well as micro/nano-electro-mechanical systems with contacts (Eriten et al., 2012), and the critical load, interference, and contact area that mark the deformation transition from purely elastic to elastic–plastic (Kagami et al., 1983; Chang et al., 1987; Majumdar and Bhushan, 1991) can be used as the normalization variables (Wang et al., 2009; Chatterjee and Sahoo, 2013) in characterizing elastic–plastic contact responses of generalized dimensionless forms Etsion et al. (2005b), Ovcharenko et al. (2008), Li et al. (2010a,b) and Shi et al. (2013). Our understanding of how geometrical and material parameters affect the critical variables such as load, interference, and contact size at the point of yield inception in a sinusoidal contact is limited since the explicit expressions have not been achieved to date. A thorough study of effects of geometrical and material properties on the onset of plasticity in normally loaded sinusoidal contacts is still missing. This work aims to extend the spherical contact to sinusoidal contact with the objective to explore effects of geometrical and material parameters on the contact variables at inception of plastic yielding.

2. Problem formulation and finite element modeling

Consider a single deformable sinusoidal asperity interacting with a rigid flat, since contact involving a rigid flat has an universal application, and the original contact between two surface topographies can be transferred to an equivalent contact between a rigid flat and a rough surface with the combined topography of the two contacting surfaces (Greenwood and Tripp, 1970; Bucher et al., 2002; Li et al., 2010a). The contact process is quasi-static without inertia effect. The deformation is mostly elastic within the scope of this work. It is assumed the deformation is localized in the vicinity of the contact (Chang et al., 1987), although effect of local contact on displacement of regions far away from the contact was observed (Pullen and Williamson, 1972; Kucharski et al., 1994). Only frictionless contact usually adopted in contact analysis (Kral et al., 1993; Yang and Kao, 1999; Mesarovic and Fleck, 2000; Mata et al., 2002; Etsion et al., 2005a; Sahoo et al., 2010; Chen et al., 2011; Celentano et al., 2012; Liu et al., 2012) is considered, since no discernible difference between frictionless and frictional contacts have been found (Mata and Alcalá, 2004). Finite element analysis of sinusoidal asperity-on-flat contact is carried out by taking into account finite strain, material orientation, and large displacement via an updated Lagrangian formulation and a corotational framework. The reference configuration is the configuration at the beginning of each increment. The sparse DSCPACK solver is used for the local resolution (Raghavan, 2002).

2.1. Material behavior

Isotropic material is considered. The stress tensor $\boldsymbol{\sigma}$ can be expressed by Celentano et al. (2012)

$$\boldsymbol{\sigma} = \mathbf{C} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_p) \tag{1}$$

where $\mathbf{C}$ is the isotropic elastic stiffness tensor, $\boldsymbol{\varepsilon}$ is strain tensor, and $\boldsymbol{\varepsilon}_p$ is plastic strain tensor.

The constitutive model of the deformable asperity is taken to follow J_2-associated flow theory with rate-independent deformation and isotropic hardening. Yielding occurs according to the von Mises criterion (Celentano et al., 2012)

$$f = \sqrt{3J_2} - \sigma_h - \sigma_y \tag{2}$$

where J_2 is the second invariant of the deviatoric part of stress tensor, $\sigma_e = \sqrt{3J_2}$ is the so-called equivalent or von Mises stress, σ_h is the isotropic hardening stress, and σ_y denotes the initial stress.

A small plastic deformation is made possible by using a linear strain hardening law in order to determine the point at which the material yields

$$\sigma_h = E_t \varepsilon_e^p \tag{3}$$

where ε_e^p is the effective plastic strain, E_t is the tangent modulus. Strain hardening does not affect the elastic deformation before plastic yielding, and thus the tangent modulus is kept constant as 1.07 GPa.