Mechanics of Materials 76 (2014) 45–63

Contents lists available at ScienceDirect

Mechanics of Materials

journal homepage: www.elsevier.com/locate/mechmat

A thermodynamically consistent growth law for collagen fiber reinforced tissues in a mixture context

Benjamin Loret [a,*], Fernando M.F. Simões [b]

[a] Laboratoire Sols, Solides, Structures, B.P. 53X, 38041 Grenoble Cedex, France
[b] ICIST and Departamento de Engenharia Civil, Instituto Superior Técnico, Universidade de Lisboa, Av. Rovisco Pais, 1049-001 Lisboa, Portugal

ARTICLE INFO

Article history:
Received 30 April 2014
Received in revised form 3 June 2014
Available online 17 June 2014

Keywords:
Biological tissues
Growth rate law
Elastic-growing solids
Dissipation inequality
Mixture

ABSTRACT

The growth of mass of the two main solid constituents of articular cartilage is addressed in the context of saturated porous media endowed with an electrical charge. As a basic constraint, the two constituents undergo the same deformation gradient but the growth and elastic (accommodating) multiplicative decompositions are a priori innate to each of them. For the collagen, five rate laws are defined to govern the evolutions of the mass, of the homeostatic surface, of the fiber network orientation distribution function, of the pyridinoline cross-link density and of the cell energy density. For proteoglycans, only the evolutions of mass, of the homeostatic surface and of the cell energy density are considered. Emphasis is laid on devising rate laws that are easily constrained to satisfy the dissipation inequality. The cell energy density plays a significant role in this formulation. For each constituent, the growth model is stress-driven: growth is controlled by the position of the stress with respect to a homeostatic surface. These surfaces and the rate laws are endowed with properties that control the boundedness of the growth process.

1. Introduction

Growth is an irreversible process, metabolism and catabolism being associated with distinct pathways. Energy provided by the metabolism is used to grow the mass and improve the mechanical properties of the tissue constituents. Heat dissipation is in some sense minimized: the tissue recovers as much energy as possible to drive the growth and stiffening/structuration processes. Indeed in contrast to damage, stiffening contributes negatively to the dissipation inequality. While there are myriads of analyses of passive processes, e.g. damage of engineering materials and bone tissues, stiffening and structuration processes have been much more scarcely addressed.

In other words, the model developed here, in the context of collagen-reinforced biological soft tissues like articular cartilages, is built so that the dissipation can only be positive or zero. The idea is that mechanisms that require energy (active processes) are scaled down if the energy available provided by passive processes is not sufficient. Thus an effort is made to delineate unambiguously the signs of the contributions to the dissipation inequality of all the processes involved in the growth of mass and in the modifications of the physical properties of the tissue.

While we will term structuration the active processes we have in mind, they present some similarities and differences with so-called self-healing phenomena. The term "self-healing" conveys the idea of repair while structuration concerns developing tissues. On the other hand, both phenomena are intended to convey the idea that energy from the surroundings in the form of heat or light is not required to achieve the process. Few cases of combined

* Corresponding author. Tel.: +33 476 825298; fax: +33 476 827000.
E-mail address: Benjamin.Loret@grenoble-inp.fr (B. Loret).

http://dx.doi.org/10.1016/j.mechmat.2014.06.001
0167-6636/

damage and self-healing processes have been analyzed in the literature:

- in engineering materials, like fiber-reinforced composites, concrete and ceramic materials, a chemical wrapped in micro-capsules and a catalyst are dispersed in the materials during the fabrication process. If the loading gives rise to diffuse microcracks, the micro-capsules are expected to open, releasing the chemical which ideally flows to cover the crack surfaces and comes in contact with the catalyst dispersed in the material. The chemical and catalyst react, using the energy dissipated by the damage process, so as to glue the crack faces (Barbero and Ford, 2007);
- for rock densification and compaction of crushed rock salt (Miao et al., 1995), the healing source is the surface energy of cracks, which is proportional to their surface.

The time scale of the reaction may be long, especially with respect to the time scale of the mechanical loading. Moreover, the efficiency of self-healing is not expected to be complete. A vascular network, bleeding and blood clotting make many biological tissues self-healing systems in the long range (Trask et al., 2007).

The structuration of the collagen network during growth of biological tissues that we have in mind goes essentially along the above description:

- indeed, if the pre-existing energy source that is used by the healing process, e.g. the chemical and catalyst concentration in the case of self-healing composite materials, did not exist, next to the mechanical load, structuration would have to rely only on the energy provided directly by the growth process;
- however, cells are seen as providing an additional energy source that decreases during the growth process and that is potentially exhausted by this process. This point of view is motivated by the measurements by Williamson et al. (2001) who show a decreasing cell mass at increasing masses of proteoglycans (PG's) and collagen.

It is of interest to contrast the phenomena of healing and *damage*, the latter corresponding qualitatively to a decrease of the elastic stiffness. *Healing is not the converse of damaging*. These considerations simply illustrate the fact that the forward and return paths of metabolic pathways include portions that are distinct and associated with dissipation.

The developments below consider a mixture where two solid constituents are bathed in an incompressible fluid. One of the constituents, namely proteoglycans, is endowed with an electrical charge which gives rise to an osmotic pressure. While the two constituents undergo the same deformation gradient, the growth and accommodating multiplicative decompositions are innate to each of them. For collagen, rate laws govern the evolutions of mass, fiber network relative volume fractions, pyridinoline cross-link density and cell energy density. For proteoglycans, only the evolutions of mass and cell energy density need to be considered. Growth is stress-driven and controlled by the position of the stress with respect to a homeostatic surface, the change of size and translation of which ensure the boundedness of the growth process and also contribute to the dissipation. For each constituent, the sum of the dissipations associated with these rate laws is constrained to satisfy the dissipation inequality.

2. Thermodynamic mixture framework

The set of species $\mathcal{K}$ of the mixture is partitioned in growing constituents $\mathcal{S}^*$ and non growing species $\mathcal{K}^*$. The growing constituents are collagen fibers and proteoglycans while non growing species include water, ionic species, and possibly macromolecules like growth factors.

The constituents of the mixture work in parallel: the velocities of all growing constituents $k \in \mathcal{S}^*$ are identical [1] while their stress contributes additively to the stress of the mixture. The deformation gradient $\mathbf{F}$ is common to all growing constituents, but the elastic-growth decomposition $\mathbf{F} = \mathbf{F}^{\mathrm{e}}_k \cdot \mathbf{F}^{\mathrm{g}}_k$ is innate to each of them as already assumed in e.g. Klisch et al. (2003). The velocity gradient is then additively contributed by an elastic and a growth component,

$$\mathbf{L} \equiv \frac{d\mathbf{F}}{dt} \cdot \mathbf{F}^{-1} = \frac{d\mathbf{F}^{\mathrm{e}}_k}{dt} \cdot (\mathbf{F}^{\mathrm{e}}_k)^{-1} + \overbrace{\mathbf{F}^{\mathrm{e}}_k \cdot \underbrace{\frac{d\mathbf{F}^{\mathrm{g}}_k}{dt} \cdot (\mathbf{F}^{\mathrm{g}}_k)^{-1}}_{\mathbf{L}^{\mathrm{g}}_{\kappa k}} \cdot (\mathbf{F}^{\mathrm{e}}_k)^{-1}}^{\mathbf{L}^{\mathrm{g}}_k}, \quad k \in \mathcal{S}^*. \tag{2.1}$$

The thermodynamic state of growing constituents is addressed in the intermediate configurations (κ) implicitly introduced by the elastic-growth decomposition. The set of independent state variables of the growing constituent k relative to these configurations,

$$\mathcal{V}_{\kappa k} = \{\mathbf{E}^{\mathrm{e}}_{Gk},\ T,\ \xi_{\kappa k}\}, \tag{2.2}$$

includes the Green strain $\mathbf{E}^{\mathrm{e}}_{Gk} = \frac{1}{2}(\mathbf{C}^{\mathrm{e}}_k - \mathbf{I})$, with $\mathbf{C}^{\mathrm{e}}_k = (\mathbf{F}^{\mathrm{e}}_k)^{\mathrm{T}} \cdot \mathbf{F}^{\mathrm{e}}_k$ the right Cauchy–Green tensor, the temperature T, and a single dissipative variable $\xi_{\kappa k}$ per growing constituent is temporarily displayed for ease of writing. It will be expanded later to represent the state of the homeostatic surface, the cell energy density, and, for collagen $k = \mathrm{c}$, an orientation distribution function and the pyridinoline cross-link density. The temperature T is common to all species as the mixture is in local thermal equilibrium.

Implicit in the choice of state variables are the following assumptions:

- the chemo-mechanical variables $\cup_{k \in \mathcal{K}^*} m^k$, typically the mass contents of non growing species, are independent of the kinematics of the growing constituents;

[1] So we have no diffusion of the growing constituents, neither inward nor outward, and the material time derivative d/dt following a growing constituent is the same for all growing constituents. The velocity of the growing constituents should be distinct from the solid skeleton to allow for diffusion of (part of) the secreted mass via water bathing the tissue.