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Analytical frequency response functions for contact of multilayered materials

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ABSTRACT

Multilayer coatings are often seen in surface engineering for surface modifications. Optimal design of the multilayered materials requires the understanding of their mechanical behaviors based on deformation and stress analyses. The frequency response functions (FRFs) of the displacement and stress fields in multilayered materials under unit normal and shear loadings are the analytical cores for solving the contact of such materials. The authors have successfully derived these functions by utilizing the Papkovich–Neuber potentials and appropriate boundary conditions. Two matrix equations containing unknown coefficients in the FRFs are established by following the structure rules, and then the closed-form FRFs written in a recurrence format are established. A fast numerical semi-analytical model based on the derived FRFs is further developed for investigating the elastic contact of multilayered materials with any desired material design.

1. Introduction

Multilayer coatings as surface modifications have been introduced to mechanical, electrical, and biomedical components for various needs. Optimal design of multilayer coatings requires the understanding of their mechanical behaviors based on deformation and stress analyses. The contact analyses of the layered materials generally require the use of numerical methods, such as the semi-analytical method (SAM) (Burmister, 1945a,b,c; Cai and Bhushan, 2005; Chen, 1971, Chen and Engel, 1972; Kuo and Keer, 1992; Leroy et al., 1989; Liu and Wang, 2002; Nogi and Kato, 1997; O'Süllivian and King, 1988; Plumet and Dubourg, 1998), the boundary element method (BEM) (Luo et al., 2000), the finite element method (FEM) (Chen and Bull, 2009; Djabella and Arnell, 1994; Gorishnyy et al., 2003; Komvopoulos, 1988; Komvopoulos and Choi, 1992; Kot et al., 2013), and the equivalent inclusion method (EIM) (Bagault et al., 2013; Chen et al., 2010). The contact of layered materials has been extensively studied over one half of a century; however, most of the researches were focused on the materials with a single-layer coating. Theoretically, the methodologies utilized in single coating system should apply to the studies of multilayered materials; however, the introduction of multicoatings significantly complicates the solution process and aggravates the computational expense.

Several finite element models have been developed to investigate the stresses and deformations of the multilayered materials (Kot, 2012; Kot et al., 2013; Lakkaraju et al., 2006). As one of the advantages of FEM, various material constitutive laws, elasto-plasticity, for an example (Kot, 2012), can be involved. However, expensive computational burden is one of its trade-offs. The boundary

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Nomenclature

a_0	contact radius in the Hertz solution of the substrate material
A_c	the area in contact
$A^{(j)}, \bar{A}^{(j)}, B^{(j)}, \bar{B}^{(j)}, C^{(j)}, \bar{C}^{(j)}$	unknown coefficients in the Papkovich–Neuber potentials in the frequency domain
$B^{(j)}_{,m}, \bar{B}^{(j)}_{,m}$	derivative of the B components with respect to m
E_j	Young's modulus of layer j [MPa]
G_j	shear modulus of layer j [MPa]
$G^{(j,j+1)}$	ratio of the shear modulus of jth layer to that of $(j+1)$th layer
$\tilde{\tilde{G}}^{u_3}(m,n,0)$	the FRF of surface displacement u_3 at node $(m,n,0)$
h_j	thickness of layer j [m]
H, H_i	surface gap and initial surface gap [m]
J_2	the second invariant of the stress deviator tensor [MPa^2]
$\sqrt{J_2}/P_0$	normalized von Mises stress
L	total number of coatings
m, n	Fourier-transformed frequency variables with respect to x and y
$p(x,y), q(x,y)$	normal and shear loadings in the spatial domain [MPa]
$\tilde{\tilde{p}}(m,n), \tilde{\tilde{q}}(m,n)$	normal and shear loadings in the frequency domain
P_0	peak contact pressure in the Hertz solution of the substrate material [MPa]
$S^{(0)}_1, S^{(0)}_2$	the S components in the surface
$S^{(j)}_1, S^{(j)}_2, S^{(j)}_3, S^{(j)}_4$	the S components at the jth interface
$S^{(j)}_a, S^{(j)}_b, t^{(j)}_0, k^{(j)}_r, D^{(j)}_r$	intermediate variables (defined in Appendix C) at the jth interface
$u^{(j)}_i$	displacement u_i in layer j [m]
x, y, z_j	Cartesian coordinates in the spatial domain
α	distance of a node (m,n) to the origin in the frequency domain
δ_{ik}	Kronecher delta
μ	friction coefficient
v_j	Poisson's ratio of the material in layer j
$\sigma^{(j)}_{ik}$	stress component σ_{ik} in layer j [MPa]
$\sigma^{(0)}_{ik}$	stress component σ_{ik} in the surface [MPa]
φ and ψ_1, ψ_2, ψ_3	Papkovich–Neuber potentials in the spatial domain
$\tilde{\tilde{\varphi}}^{(j)}, \tilde{\tilde{\psi}}^{(j)}_1, \tilde{\tilde{\psi}}^{(j)}_3$	Papkovich–Neuber potentials in the frequency domain in layer j
ω	normal rigid body approach [m]

Special marks

$\approx$	double Fourier transforms operations
$\circ$	direct multiplication in the frequency domain
FT_{xy}	double Fourier transform about x and y
$IFFT$	inverse fast Fourier transform

Superscripts or subscripts

$j = 1,\ldots,L$	layer or interface number
m	derivative with respect to m

element method was introduced to analyze the interfacial stress of a multi-coating system, where a two-dimensional plane strain problem was solved with the aid of integrals (Luo et al., 2000). The equivalent inclusion method can only consider the influence of coatings in the computation domain instead of infinitely spreading layers (Chen et al., 2010).

With the semi-analytical methods, the analytical relationships between responses (displacements and stresses) and excitations (surface normal and shear tractions) have to be solved through use of the integral transform theory, which maps a complicated spatial problem into an equivalent, usually much simpler problem in the transformed frequency domain. Such problems involve the use of the Papkovich–Neuber potentials or the Galerkin vector potentials, where the potentials functions in these equations can be written in integral transforms. Double Fourier series or integrals are usually utilized for rectangular domains and Hankel transforms for circular domains (Keer, 2008). The general theories of the stresses and displacements of layered soil systems were structured by Burmister (1945a,b,c) with the aid of the Bessel type of stress functions for analyzing the contact under prescribed axisymmetric surface normal loading. Both fully bonded interface and frictionless interfaces were studied. Chen (1971) and Chen and Engel (1972) extended the applications to both axisymmetric and non-axisymmetric normal surface loading. The displacement and stresses were expressed in terms of two harmonic functions in the form of Fourier-type integrals. Boundary and continuity conditions were utilized to solve for the unknown coefficients. The final elastic solutions in the spatial domain were obtained by applying the inverse Fourier transform, requiring numerical quadrature for the integrand evaluation. O'Süllivan and King (1988) studied the contact of bi-layered materials under normal and shear loads. With the aid of the Papkovich–Neuber potentials in the frequency domain and appropriate boundary conditions, a linear system of equations were set up for solving the unknown coefficients in the displacement and stresses expressions. The closed-form equations were thus explicitly derived, and the solutions transferred back to the spatial domain via numerical inverse transformation. Kuo and Keer (1992) studied the contact of multilayered transversely isotropic materials under normal and tangential loading via the Hankel transform. The multilayer coatings were handled with a greatly reduced effort by introducing the propagator matrix and continuity conditions at each interface.

FFT techniques were applied to contact mechanics by Ju and Farris (1996) to achieve significant increase in computation speed by using the discrete Fourier transform instead of the above-mentioned continuous integral transforms. Nogi and Kato (1997) applied the FFT technique and