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Indentation on two-dimensional hexagonal quasicrystals

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ABSTRACT

This paper is concerned with contact problem for a half-space of two-dimensional hexagonal quasi-crystal punched by three common indenters (cylindrical flat-ended, conical and spherical punches). Based on the Green's functions for the half-space subjected to an external phonon source exerted on the surface, the superposition principle is applied to constructing the boundary integral equation. Relations between the indentation force and the penetration depth, and the indentation stiffness constants are explicitly obtained for these three indenters. Complete and exact fields in the half-space are given in terms of elementary functions. The present theoretical solutions can not only serve as benchmark for computational contact mechanics, but also apply to guiding future indentation experiments.

1. Introduction

Since the discovery of quasi-crystals (QCs) by Shechtman et al. (1984), its mechanical behaviors have gradually been an important branch of condensed matter physics (Fan, 2011). Owing to the special arrangement of atoms, QC bodies have many desirable properties, for instance, lower friction coefficients, lower adhesion, higher wear resistance and lower porosity in contrast with the media of traditional crystals (Dubois et al., 1991; Kenzari et al., 2012). This enables QCs to enjoy a promising prospect in numerous engineering fields (Belin-Ferre et al., 2000; Brunet et al., 2000).

In the past three decades, a great deal of research efforts have been made to study the crack and dislocation problems (De and Pelcovits, 1987; Ding et al., 1995; Fan and Mai, 2004; Fan, 2011); for example, Wang (2004), taking advantage of complex variable method, characterized the interaction between a half-infinite plane crack and a line dislocation in decagonal QCs in terms of local stress intensity factors, energy release rate (ERR) at the crack tip and the Peach-Koehler force acting on the line dislocation. By virtue of Stroh formalism, steady propagation of a straight dislocation in an unbounded elastic quasi-crystal with five-fold symmetry was analyzed by Radi and Mariano (2011). In the framework of two-dimensional (2D) elasticity of QCs, Gao et al. (2011) used complex potentials to seek the explicit expressions for stress intensity factors, crack open displacements and strain energy release rate, for an elliptical hole or crack. Fan et al. (2012) presented general fracture theory of QCs concerning with linear, nonlinear and dynamic fracture problems. Recently, Li (2013) extended the potential theory method developed by Fabrikant (1989, 1991) to thermo-elasticity of one-dimensional (1D) hexagonal QCs and derived the non-axisymmetric fundamental solutions for an infinite space weakened by a penny-shaped or half-infinite plane crack. Comprehensive literature review on the developments relative to the mechanics of QCs is far beyond the scope of the present paper. The reader interested in dislocation,

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inclusion and crack problems, can refer to the monograph (Fan, 2011) and the review paper (Fan and Mai, 2004).

As pointed out by Fan (2011) and Radi and Mariano (2011), the data for the material constants involved in constitutive relations, especially those characterizing the phonon–phason interaction activity, are very rare. To fill the gap, various methods including Monte Carlo simulation (Strandburg et al., 1989; Tang, 1990), relaxation simulation (Zhu and Henley, 1999; Koschella et al., 2002), transfer-matrix method (Newman and Henley, 1995), diffuse scattering (Dubois et al., 1991; de Boissieu et al., 1995; Létoublon et al., 2001) and X-ray diffraction measurements (Edagawa, 2005) have been reported.

Apart from the foregoing approaches, indentation technique and atomic force microscopy (AFM) are widely employed to characterize the mechanical behavior of QCs (Azhazha et al., 2005; Mukhopadhyay et al., 2006). This technique evidently rely on the contact solutions, in the framework of elasticity of QCs owing to the phonon–phason coupling behavior. Furthermore, during the indentation test, the resolution of microscopy setting depends on deep insights in the pressure-response relation (Karapetian and Kalinin, 2011).

According to the literature survey, there are quite a few records concerning the contact analyses for QC body. Peng and Fan (2001), making use of Fourier series expansion and Hankel transform, derived a general solution in an integral form for a half-space of one-dimensional hexagonal QC, indented by a cylindrical punch. Following a similar method, Zhou et al. (2002) investigated an axisymmetric contact problem for cubic QCs, and presented the distribution of the generalized stresses underneath the rigid indenter. In these two articles (Peng and Fan, 2001; Zhou et al., 2002), however, the indentation force versus indentation depth, which is of high importance in experiment, is not given. Furthermore, the distributions of phonon and phason fields in the half-space are not discussed either. Recently, Wu et al. (2013) examined the axisymmetric contact problem for an half-infinite space of 1D hexagonal QC punched by three indenters (flat-ended cylinder, cone and sphere) and derive the intrinsic links between the indentation force and penetration depth.

Atoms of two-dimensional (2D) QCs are arranged periodically in one direction and aperiodically in the plane normal to previous direction, which is quite different from those of 1D QCs. As a result, the constitutive laws for 2D QCs differ from those for 1D ones. In fact, the generalized Hook's laws for the former are more complicated than those for the latter (Hu et al., 2000; Fan, 2011): the strain originating from two phason displacements are introduced for 2D QCs, and hence more material constants are involved in the generalized stress–strain relations.

This paper aims to present general theory on indentation over a half-space of 2D hexagonal QCs indented by three types of typical punches. Corresponding half-space Green's functions to an external concentrated force in the phonon field are derived by analogy with the methods proposed by Ding et al. (1997) for piezoelectric media. Boundary integral equation governing the contact problem is derived by superposing the Green's function, and is solved by virtue of potential theory method initiated by Fabrikant (1989, 1991). Explicit expressions for indentation force and penetration depth are derived for these three indenters. The indentation stiffness constant, an intrinsic parameter in experimental test, is uniformly given by a single expression for the indenters. The coupling phonon–phason fields in the half-space are obtained in terms of elementary functions. Numerical calculations are carried out to discuss various issues of special interests.

2. Basic equations and fundamental solutions

2.1. Basic equations

Consider a two-dimensional (2D) hexagonal Quasi-crystal body, whose atoms are arranged quasi-periodically in the $r-\theta$ plane, and periodically in the z-direction, by referring to the cylindrical coordinate system (r,θ,z). The constitutive relations of the QCs read (Hu et al., 2000; Gao and Zhao, 2009)

$$\begin{aligned}
\sigma_{rr} &= c_{11}\varepsilon_{rr} + c_{12}\varepsilon_{\theta\theta} + c_{13}\varepsilon_{zz} + R_1\omega_{rr} + R_2\omega_{\theta\theta} \\
\sigma_{\theta\theta} &= c_{12}\varepsilon_{rr} + c_{11}\varepsilon_{\theta\theta} + c_{13}\varepsilon_{zz} + R_2\omega_{rr} + R_1\omega_{\theta\theta} \\
\sigma_{zz} &= c_{13}(\varepsilon_{rr} + \varepsilon_{\theta\theta}) + c_{33}\varepsilon_{zz} + R_3(\omega_{rr} + \omega_{\theta\theta}) \\
\sigma_{\theta z} &= \sigma_{z\theta} = 2c_{44}\varepsilon_{\theta z} + R_4\omega_{\theta z} \\
\sigma_{rz} &= \sigma_{zr} = 2c_{44}\varepsilon_{rz} + R_4\omega_{rz} \\
\sigma_{r\theta} &= \sigma_{\theta r} = 2c_{66}\varepsilon_{r\theta} + R_6(\omega_{r\theta} + \omega_{\theta r})
\end{aligned} \tag{1a}$$

$$\begin{aligned}
H_{rr} &= R_1\varepsilon_{rr} + R_2\varepsilon_{\theta\theta} + R_3\varepsilon_{zz} + K_1\omega_{rr} + K_2\omega_{\theta\theta} \\
H_{r\theta} &= 2R_6\varepsilon_{r\theta} + K_3\omega_{r\theta} + K_6\omega_{\theta r} \\
H_{rz} &= 2R_4\varepsilon_{zr} + K_4\omega_{rz}
\end{aligned} \tag{1b}$$

$$\begin{aligned}
H_{\theta r} &= 2R_6\varepsilon_{r\theta} + K_6\omega_{r\theta} + K_3\omega_{\theta r} \\
H_{\theta\theta} &= R_2\varepsilon_{rr} + R_1\varepsilon_{\theta\theta} + R_3\varepsilon_{zz} + K_2\omega_{rr} + K_1\omega_{\theta\theta} \\
H_{\theta z} &= 2R_4\varepsilon_{\theta z} + K_4\omega_{\theta z}
\end{aligned} \tag{1c}$$

where

$$2c_{66} = c_{11} - c_{12}, \quad K_6 = K_1 - K_2 - K_3, \quad 2R_6 = R_1 - R_2 \tag{2}$$

σ_{ij} and H_{ij} are the stress components in phonon and phason fields, ε_{ij} and ω_{ij} denote the phonon and phason strain components; c_{ij}, K_i and R_i represent the phonon, phason and phonon–phason coupling elastic constants, respectively.

The strain components are expressed in terms of the phonon displacements (u_i) and phason ones (w_i) as

$$\begin{aligned}
&\varepsilon_{rr} = \frac{\partial u_r}{\partial r}, \quad \varepsilon_{\theta z} = \varepsilon_{\theta z} = \frac{1}{2}\left(\frac{\partial u_\theta}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \theta}\right) \\
&\varepsilon_{\theta\theta} = \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r}, \quad \varepsilon_{rz} = \varepsilon_{zr} = \frac{1}{2}\left(\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z}\right) \\
&\varepsilon_{zz} = \frac{\partial u_z}{\partial z}, \quad \varepsilon_{r\theta} = \varepsilon_{\theta r} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right)
\end{aligned} \tag{3a}$$

$$\begin{aligned}
&\omega_{rr} = \frac{\partial w_r}{\partial r}, \quad \omega_{r\theta} = \frac{1}{r}\frac{\partial w_r}{\partial \theta}, \quad \omega_{rz} = \frac{\partial w_r}{\partial z} \\
&\omega_{\theta r} = \frac{\partial w_\theta}{\partial r}, \quad \omega_{\theta\theta} = \frac{1}{r}\frac{\partial w_\theta}{\partial \theta} + \frac{w_r}{r}, \quad \omega_{\theta z} = \frac{\partial w_\theta}{\partial z}.
\end{aligned} \tag{3b}$$

In the absence of body forces, the generalized equilibrium equations for 2D hexagonal quasi-crystals are of the following form