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Study of the concept of representative strain and constraint factor introduced by Vickers indentation

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ABSTRACT

The application of the concept of the representative strain is often used in the stress–strain curve determination from indentation test because it can significantly simplify the analysis of the indentation response. A new methodology for determining the representative strain for Vickers indentation is presented in this article. Following a procedure based on finite element simulations of indentation of elastoplastic materials, two representative strains are defined: the representative strain characteristic of the mean pressure and the representative strain characteristic of the Martens hardness or the indentation loading curvature. The results obtained from this methodology show that there is no universal value of representative strain independent of the mechanical parameters of materials indented by Vickers indentation. It is also shown that the representative strain, obtained by Vickers indentation is much lower when it is obtained from the relationship between the applied force and the penetration depth, *F-h*, rather than from the relationship between the applied force and the contact radius, *F-a*. The values of the calculated representative strains show that simultaneous measurement of relationships *F-a* and *F-h* make it possible to characterize the hardening law with two unknown parameters by Vickers indentation.

1. Introduction

Indentation tests can be used not only for the evaluation of hardness, but also in the determination of other mechanical properties such as Young's modulus and stress–strain curves. The application of the concept of the representative strain can significantly simplify the analysis of the indentation response and was often used in the stress–strain curve determination from indentation test (Tabor, 1951; Giannakopoulos and Suresh, 1999; Venkatesh et al., 2000; Dao et al., 2001; Chollacoop et al., 2003; Bucaille et al., 2003; Kermouche et al., 2005; Ogasawara et al., 2005; Cao and Huber, 2006; Antunes et al., 2007; Kermouche et al., 2008). In the case of conical indentation, the representative strain, ε_R, is independent of the size of the indentation and depends on the half apex angle of the indenter, θ, which is equal to 70.3° for a conical indenter equivalent to the Vickers indenter. The studies performed on the representative strain in Vickers indentation can be divided into two groups, a first group which is based on the Mean Pressure (Tabor, 1951; Samuels and Mulhearn, 1957; Giannakopoulos et al., 1994; Chaudhri, 1998; Giannakopoulos and Suresh, 1999; Venkatesh et al., 2000; Mata et al., 2002; Kermouche et al., 2005; Kermouche et al., 2008; Branch et al., 2010) and a second group which is based on the Martens hardness (Dao et al., 2001; Bucaille et al., 2003; Chollacoop et al.,2003; Ogasawara et al., 2005; Cao and Huber, 2006; Antunes et al., 2007). The first group of studies concerns the definitions of the representative strain, which can lead to a relationship between a constant, C_F, called "the constraint factor", the Hardness, H, and the flow stress, σ_R, at a representative value of the plastic strain, ε_R *i.e.*:

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$$C_F = \frac{H}{\sigma_R} \quad (1)$$

In this relationship, H corresponds to the mean contact pressure, which is calculated from the diameter of the contact circle at full load (assumed to be equal to the diameter of the residual impression in the surface).

The second group of studies concerns the definitions of the representative strain, which can lead to a relationship between the reduced Young's modulus, E^*, the indentation loading curvature, C_L, and the representative stress, σ_R, *i.e.* (Dao et al., 2001):

$$C_L = \frac{F}{h^2} = \sigma_R \Pi_1 \left(\frac{E^*}{\sigma_R}\right) \quad (2)$$

In this relationship, the determination of the loading curvature, C_L leads to the determination of the "Martens" hardness, HM, which is equal to the following expression in the case of Vickers indentation:

$$HM = \frac{F}{26.43\, h^2} = \frac{C_L}{26.43} \quad (3)$$

The concept of representative strain was first introduced by Tabor (1951) to relate its corresponding representative stress to the Mean Pressure value. Tabor proposed, from experiments on essentially two materials, mild steel and copper, that the representative strain is equal to 0.08 in the case of Vickers indentation. This value is obtained so that the ratio of the Mean Pressure, H, to the corresponding representative stress, σ_R is equal to 3.3 (value previously determined from experiments performed on work-hardened metals) (Tabor, 1951). The value of 0.08 proposed by Tabor is similar to the value of 0.07 reported by Samuels and Mulhearn (1957) in the case of Vickers indentations in annealed 70:30 brass. This proposition is also close of the numerical results obtained by Mata et al. (2002) for conical indentation of elastic plastic materials with various Young modulus, E, Yield stress, σ_y, and hardening exponent n, i.e. $\varepsilon_R = 0.1$. With this value of representative strain, the ratio of the hardness, H to the corresponding representative stress, $\sigma_{R,}$ was found equal to 2.7. For Mata et al. (2002), the accuracy of Tabor's equation is limited to the fully plastic contact regime. As long as this regime prevails, Tabor's equation, i.e. $H/\sigma_{(\varepsilon=0.08)} = 3.3$, is found to be extremely accurate as hardness values estimated by this equation.

From an experimental investigation of the surface and subsurface strain hardening around Vickers indentations in annealed copper, it was determined that the maximum plastic strain occurs in a subsurface region close to the indentation tip where the estimated plastic natural strain is in the range from 0.25 to 0.36 (Chaudhri, 1998). Srikant et al. (2006) found similar values of maximum strain for similar experimental conditions (maximum plastic strain in the range between 0.22 and 0.31). Chaudhri (1998) suggest that the equivalent strain associated with a relatively large Vickers indentation should be 0.25–0.36 for annealed metals having a power law uniaxial stress vs strain relationship. Moreover, finite element computations using a conical indenter equivalent to the Vickers indenter ($\theta = 70.3°$) show that the equivalent plastic strain within a 7075-T651 aluminum exceed 15% in the majority of the volume directly beneath the indenter. Giannakopoulos et al. (1994), Giannakopoulos and Suresh (1999) and Venkatesh et al. (2000) used a "characteristic strain" of 29–30% within the context of their formulation. Giannakopoulos and Suresh (1999) suggested that the region of material experiencing strains beyond 29% under the indenter exhibits plastic "cutting" characteristics and may be modeled using slip line theory. These values are considerably higher than the value of 0.08 proposed by Tabor (1951) almost 60 years ago. Tabor's proposal was based on a fundamental assumption according to which the ratio of Vickers hardness to uniaxial Flow stress, corresponding to any prior strain plus an additional strain introduced by the indentation process, should be universally constant and equal to 3.3. This original definition does not represent any apparent physical transition in mechanical response. Moreover, this assumption has not been fully justified so far, experimentally or theoretically. Chaudhri (1998) also shows that there is very little difference between choosing $\varepsilon_r = 0.08$ and $\varepsilon_r = 0.2$ as far as the ratio of the Vickers hardness to the flow stress is concerned. For Chaudhri (1998), $\varepsilon_R = 0.08$ is not a unique value of the equivalent strain introduced by a Vickers indentation. He suggests that a better choice of the equivalent strain should be related to the maximum strain produced in the deformed zone. For Branch et al. (2010), the best choice is rather the volume average plastic strain within the plastic zone of Vickers indentation. Some authors have concluded that the Mean Pressure does not depend on a unique representative strain. Dugdale (1958), who investigated the stress–strain curves and Vickers hardness of a number of metals, alloys and nylon, has proposed that the stress–strain curves up to a strain of 0.15, and not just the stress corresponding to a single value of strain, are relevant in predicting their Vickers hardness values. For Larsson (2001), at indentation of rigid-plastic power-law materials, the hardness is well-described by a single representative strain level in the spirit of Tabor. In this case, the Vickers Hardness calculated with the constant values $C_F = 2.55$ and $\varepsilon_R = 0.18$ or $C_F = 2.8$ and $\varepsilon_R = 0.15$ are in fairly good agreement with the numerical results. In a general situation, i.e. at indentation of materials with more irregular stress–strain relations, Larsson (2001) found that the concept of a single representative strain is no longer valid. For this general situation, an alternative two-parameter description of the Mean Pressure is suggested with the two parameters corresponding to the stress levels at 2% and 35% plastic strain. To conclude, the different studies on a material-dependent representative plastic strain valid in the conversion of flow stress to Mean Pressure suggest that there may not be a universal value for the equivalent strain introduced by a Vickers indentation.

In the case of "Martens" hardness, Dao et al. (2001) shows that the value of the "representative" strain depends on the choice of functional definitions that is used to relate certain indentation parameters to certain mechanical properties. Using dimensional analysis, a set of new universal dimensionless functions was constructed to characterize instrumented sharp indentation. Based on this dimensional analysis, a representative plastic strain