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Development of plastic nonlinear waves in one-dimensional ductile granular chains under impact loading

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ABSTRACT

A modified split Hopkinson pressure bar (SHPB) was used to load one-dimensional granular chains of metallic spheres under impact loading rates. These homogeneous chains, comprised of brass spherical beads ranging from a single sphere to a chain of sixteen, are of interest because of their unique wave propagation characteristics. In the elastic range, for loads around 10 s of N, nonlinear elastic solitary waves have been observed to form. In this work, loading magnitudes spanning from 9 kN to 40 kN – considerably higher than most previous works on these systems which have been conducted in the elastic regime – cause the granular chains to severely deform plastically. The aim of this study is to identify whether a nonlinear solitary-type wave will be generated under such high load levels, and if so, under what conditions (*e.g.*, chain length, load level, *etc.*) it will do so. The propagating pulse was found to assume a distinctive shape after travelling through five beads, similar to the elastic case where solitary waves are realized with a traveling wavelength of five bead diameters. The wave speed of the plastic pulses observed here was seen to depend on maximum force, indicating that indeed it is a nonlinear wave in nature and is comparable to the elastic solitary wave. Locally, the plastic dissipation at every contact point through the chains was studied by measuring the residual plastic contact area. It was found that after the formation of the plastic nonlinear solitary wave had occurred there is also decreasing plastic deformation along the chain length except at the end beads in contact with the SHPB, which rebound into the SHPB bar causing larger plastic dissipation locally. To our knowledge this research is the first effort to investigate in detail the development and evolution of solitary-like waves in the plastic regime and will form the basis of future work in this area.

1. Introduction

A granular chain, which in its simplest form can be thought of as a series of spheres in contact, refers to a one-dimensional (1D) array of macroscopic particles under physical contact. Solitary waves can occur in elastic granular chains under dynamic loading, and have been extensively studied both numerically and experimentally in elastic chains, *e.g.*, (Sen et al., 2008). Solitary waves result from the nonlinear nature of the force–displacement relation governing the contact between two elastic particles. However, unlike the elastic case, little work has been performed in the plastic regime of grain deformation, even in the simplest case of a 1D chain. In the plastic regime, two types of nonlinear response are possible: (i) the nonlinear contact between neighboring grains which is responsible for the solitary wave generation in the elastic case, and (ii) the material nonlinearity introduced by plasticity which will inevitably occur at the high stress concentration region of the contact points. The interplay between these two nonlinear phenomena forms the focus of the present

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effort which aims to gain a better understanding of whether plastic solitary-like waves form in 1D ductile chains and what governs their formation and propagation (*e.g.*, chain length, applied load, bead material *etc.*).

The contact between two elastic spheres is commonly modeled as the Hertz contact problem (Coste and Gilles, 1999; Hertz, 1896). The formulation of the Hertz contact between spheres, detailed in Johnson (1985), is based on the following three assumptions: (i) the radii of curvature of the contacting bodies are large compared to the size of the contact region, allowing each surface to be treated as an elastic half-space, (ii) the dimensions of each body are large compared to the radius of the contact region allowing the indentation stresses and strains to be independent of the evolving geometry, and (iii) the contact is frictionless. In such a case, the elastic force F_o exerted on two identical spheres in contact is related to the distance of approach δ of their centers by

$$F_o = \frac{2\bar{E}}{3}\sqrt{\frac{a}{2}}\delta^{3/2}, \text{ with } \bar{E} \equiv \frac{E}{1-\nu^2}, \tag{1}$$

where a is the sphere radius, E is the Young's modulus, and ν is the Poisson's ratio (Coste and Gilles, 1999). When d, the distance between the centers of two spheres, becomes greater than $2a$, no force is exerted, as shown schematically in Fig. 1(a) (δ is taken to be positive in compression). Fig. 1(b) shows a visual representation of δ and d.

It has been shown that a chain of spheres will generate nonlinear stress waves resulting from this Hertzian contact (Lazaridi and Nesterenko, 1985) provided the deformation time is larger than the characteristic period of particle oscillation (Nesterenko, 2001). As a consequence of the interaction law shown in Fig. 1(a), which exhibits a "sonic vacuum" (Gavrilyuk and Nesterenko, 1993; Nesterenko 1992) since it is not linearizable even for small magnitudes of force, a solitary wave will be generated in the granular chain. However, this law has a natural limit caused by possible plastic flow of the material near the contacts, which can occur for fairly small loads (Johnson, 1985; Wang et al., 2013). This significantly limits the extent of elastic behavior during solitary wave propagation. From the extensive study of 1D elastic systems certain characteristics of solitary waves have become known: (i) the solitary wave maintains its shape as it propagates through the chain, (ii) the wave propagates with a width of approximately five particle diameters in the chain, and (iii) the speed of the wave is a function of the maximum force, material properties of the particle, and the size of the particle. The velocity of propagating waves in the elastic 1D system scales as $F_m^{1/6}$, where F_m is the maximum force in the wave. The exact solution for the continuum approximation of this solitary wave shown in Eq. (2) contains the shape of a $\cos^4$ function and a compact support equal to five particle diameters (Coste et al., 1997; Nesterenko and Lazaridi, 1987; Nesterenko, 1984; Sadd et al., 1997; Sen et al., 2008)

$$v(t) = \frac{du(t)}{dt} = \frac{25}{16}\cos^4(2t/\sqrt{10}) \tag{2}$$

In the strongly nonlinear regime (Nesterenko, 1984), the speed of the solitary wave, V_s, becomes (Daraio et al., 2006)

$$V_s = 0.6802\left(\frac{2E}{a\rho^{\frac{3}{2}}(1-\nu^2)}\right)^{1/3} F_m^{1/6} \tag{3}$$

However, this relation has not been verified for higher dimensions due to geometrical effects (Goddard, 1990) although recent experiments and simulations suggest that it holds in 2D as well (Leonard et al., 2011; Awasthi et al., 2012). The high forces applied to the granular chains in this paper also induce a large amount of plasticity, which produces an initial monotonic shock profile instead of an oscillatory wave. A similar effect was observed in the analysis of a granular chain with viscous dissipation where the dissipation was modeled by adding a term to the nonlinear spring stiffness that depended on relative bead center-to-center velocity (Herbold and Nesterenko, 2007).

Further studies on the dynamic behavior of granular media have been conducted by Sadd et al. (1997), Shukla et al. (1993, 1992, 1991, 1990) who used dynamic photoelasticity, simulations, and numerical models to study wave propagation in granular chains made of polymeric disks. It was found that at a given contact, the load from the propagating wave increased from zero to a peak value, and then gradually decreased to zero. The peak loads decreased by 20% as the wave traveled through five disk diameters, which is considerably higher than the two percent drop observed in a bar of uniform material over the same distance. Additional experiments conducted by Coste

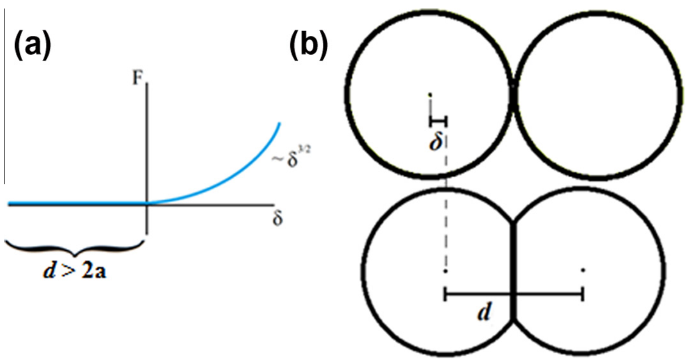

Fig. 1. (a) Nonlinear force–displacement contact relation between two spheres. (b) Diagram of two spheres before and during compression.