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On the contact mechanics of a rolling cylinder on a graded coating. Part 1: Analytical formulation

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ABSTRACT

In this paper, the fully coupled rolling contact problem of a graded coating/substrate system under the action of a rigid cylinder is investigated. Using the singular integral equation approach, the governing equations of the rolling contact problem are constructed for all possible stick/slip regimes. Applying the Gauss–Chebyshev numerical integration method, the governing equations are converted to systems of algebraic equations. A new numerical algorithm is proposed to solve these systems of equations. Both the coupled and the uncoupled solutions to the problem are found through an implemented iterative procedure. In Part I of this paper, the analytical formulation of the rolling contact problem and the discretization of the governing equations are introduced for all assumed stick/slip regimes. A detailed discussion of the proposed numerical algorithm, the iteration procedure and the numerical results, obtained using the analytical formulation, are given in Part II.

1. Introduction

Achieving the precise design goals of the modern industrial systems such as automotive and aeronautics demands the invention of new materials which are capable to resist severe loading conditions as well as highly corrosive environments. Functionally Graded Materials (FGMs) are an appropriate candidate for such critical situations. The graded materials are composites constructed by varying the micro-structure from one material to another one with a specific gradient. These materials have significant properties which can be used to avoid corrosion, fatigue, fracture and stress corrosion cracking.

Nowadays, the graded materials are commonly used in practical applications such as tribological systems, computer circuit boards and biomechanics (Enomoto and Yamamoto, 1998; Holmberg et al., 1998; Donnet and Erdemi, 2004; Pindera et al., 1996; Pompe et al., 2003). Ganesh et al. (2005) reported that a stiffness-graded plate facilitates the callus formation and improves the bone healing characteristics in comparison with a traditional stainless-steel plate. The results given by Ke et al. (2008) indicate that graded piezoelectric materials can improve the resistance to contact damage and electricity induced failure of the surface as in actuators, sensors, transducers, and micropower generators. Dag et al. (2009) proved that the lateral gradation of material can suppress the driving forces that affect the formation of herringbone cracks at brittle surfaces. This outcome is quite important for biomechanics behavior of bone-fracture fixation. Peng et al. (2009) observed that the shakedown capability of FG structures can be enhanced by optimizing the properties of the attendant constituents and the distributions of overall material properties. The analysis conducted by Yalamanchili and Sankar (2012) reveals that for a given impact energy in low-velocity impacts, the maximum stresses and strains are significantly lower in FG beams which is suitable in low-velocity foreign object impact. Zhang (2012) showed that controlled gradients of elastic modulus significantly increase the resistance of the surface against partial cone fracture which signifies FGMs as an outstanding candidate in dental restorations.

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Due to several benefits of the graded materials mentioned above, extensive technical studies have been conducted in order to examine the behavior of the these materials under different loading conditions. Bakirtas (1980) solved the normal indentation of a non-homogenous substrate under the plane strain and the plane stress conditions. Later, Delale and Erdogan (1983, 1988) analyzed the interface crack problem in a non-homogeneous half-space. Giannakopoulos and Suresh (1997a,b) formulated the point force and axisymmetric indentations of an elastic substrate with power law as well as exponentially varying Young's modulus. They found the displacement and stress fields in an analytical framework. A comprehensive study on the sliding contact problem of functionally graded coatings was conducted by Guler and Erdogan (2004, 2007). Applying the Fourier transform, they expressed the governing equations of an exponentially graded coating/substrate system in terms of coupled singular integral equations. Ke and Wang (2007a) utilized a piece-wise linear model to solve the sliding contact problem of a graded coating for various stamp profiles.

Besides the sliding contact problems and the fracture analyses, the partial slip and the fretting contact problems have drawn wide attention from the materials research community. The fretting contact is an important issue in the fatigue life of the tribological components and may result in surface damage and cracking. According to the experimental analyses, soft materials often exhibit higher susceptibility to fretting than hard materials of a similar type (Neyman and Olszewski, 1993). Giannakopoulos and Pallot (2000) considered the partial slip and the oscillating contact problem of a power-law graded half-plane. According to their study, the hardening substrates give rise to a larger stick zone in comparison with the homogeneous materials. The fretting contact problem of a graded coating was solved by Ke and Wang (2007b,c). Their results indicate that proper variation of the material property gradient can improve the resistance to the fretting contact damage. The partial slip contact problem of a rigid punch indenting a fully graded half-plane was addressed by Elloumi et al. (2008, 2010). Their study on the coupling effect showed that the uncoupled solution predicts a greater value for the stick zone size with respect to the coupled solution. Recently, Ke and Wang (2010) investigated the fretting contact problem of two elastically graded coatings. Their analysis showed that a softening coating leads to a decrease in the surface normal and shear stresses which may have a bearing on the fretting resistance.

The rolling contact phenomenon occurs in various load transferring components such as bearings, camshafts and followers, gears, continuously variable transmissions (CVT) and metal working processes. A critical issue about the rolling contact elements is the life time assessment due to the rolling contact fatigue (RCF). RCF is ascribed to the surface damages arisen from the repeated stresses when the contacting bodies roll on each other. These surface damages include spalling, delamination, wear, crack nucleation and propagation. The surface quality and stress distribution mainly affect the RCF behavior. On the other hand, the ceramic coatings (e.g. silicon nitride) and FGM coatings (e.g. WC-NiCrBSi) offer several advantages with respect to the traditional coatings under the rolling contact condition. Hence, several experimental studies have been conducted to investigate the rolling contact fatigue response of such materials (Stewart et al., 2004, 2005). Used as coatings, the graded materials can enhance the surface properties and alter the stress distribution by appropriately adjusting the elastic gradient through the coating thickness.

In the case of the rolling motion, it has been shown (Johnson, 1985) that some particles adhere inside the contact region while others may slip relative to each other. The difference between the tangential strains of the contacting particles within the adhesion area is known as the creep ratio. This creepage leads to creep forces which may play an important role in the dynamics, stability and the steering of the railroad vehicles. Due to the practical importance of the creep forces, several models have been developed for the determination of these forces (Shabana et al., 2008). According to such models, the relation between the creep forces and the creepages can be either linear or non-linear. For example based on the Kalkers linear theory, an increase in the creep ratio will directly intensify the creep forces. On the other hand, the concept of material property grading enables the material tailoring for achieving the desired mechanical properties. Hence, it would be quite interesting to explore the effect of material gradual variation on the dynamic characteristics of the rolling contact problem such as creep ratio.

The history of the rolling contact problem for the homogeneous materials is quite old (Reynolds, 1876; Carter, 1926). Bentall and Johnson (1967) analyzed the micro-slip phenomenon in the rolling contact problem of two dissimilar elastic cylinders. Kalker (1971a,b) proposed a variational approach and solved the steady state as well as the transient contact problems of the rolling cylinders. Later, Nowell and Hills (1988a,b) solved the rolling contact problem of two dissimilar and tyred cylinders using a piece-wise linear distribution for the surface contact tractions. Recently, Reina et al. (2010, 2011) solved the interfacial slip problem of a homogeneous tyred cylinder under the rolling contact condition using the distributed dislocation method. Although there are comprehensive investigations on various rolling contact problems for homogeneous materials, to the best of authors knowledge, the fully coupled solution of the contact problem of a rolling cylinder on a graded coating is not available in the open literature. Fig. 1(a) demonstrates a graphical abstract of the relevant studies on the sliding, partial slip and rolling contact problems investigated heretofore. Hence, the aim of this work is to study the mentioned problem in order to investigate the effect of material property grading on the surface stress components as well as the creep ratio. This paper is an extension of the authors recent work (Guler et al., 2012) which examined the uncoupled solution of the rolling contact problem of a graded coating/substrate system by using the Goodman approximation. They also assumed that a central stick zone along with the two slip zones of the same sign controls the contact region. In contrast, the present study considers a fully coupled rolling contact problem of a graded coating/substrate system for all