Mechanics of Materials 68 (2014) 217–227

Contents lists available at ScienceDirect

Mechanics of Materials

journal homepage: www.elsevier.com/locate/mechmat

An anisotropic discrete fiber model with dissipation for soft biological tissues

Cormac Flynn [a,*], M.B. Rubin [b]

[a] School of Engineering, Waikato Institute of Technology, Hamilton, New Zealand
[b] Faculty of Mechanical Engineering, Technion – Israel Institute of Technology, 32000 Haifa, Israel

ARTICLE INFO

Article history:
Received 24 November 2012
Available online 8 August 2013

Keywords:
Anisotropy
Discrete fiber model
Dissipation
Soft tissue mechanics

ABSTRACT

Nonlinear three-dimensional constitutive equations are developed for analyzing inelastic effects that cause dissipation in biological tissues. The model combines a structural icosahedral model of six discrete fiber bundles with a phenomenological model of the inelastic distortional deformations of the matrix containing the fibers. The inelastic response of the matrix is characterized by only three material parameters, which can be used to model both rate-independent and rate-dependent response with a smooth elastic-inelastic transition. Also, a robust, strongly objective scheme is discussed, which allows the model to be easily implemented into finite element computer codes. Examples show that the model predictions compare well with experimental data for the nonlinear, anisotropic, inelastic response of a number of tissues. Specifically, the model simulated the biaxial stretching of rabbit skin with an error of 15.7%, stress relaxation of rabbit skin with an error of 17.2%, simple shear of rat septal myocardium with an error of 21.6%, and uniaxial stress in compression of monkey liver with an error of 8.3%.

1. Introduction

Constitutive models for soft biological tissues can be used for a variety of applications, which include: design of biomedical devices, surgical simulators, assessment of the relative effectiveness of specific surgical procedures, simulations of crash dummies, and even for improving realism of animation in movies. Collagen and elastin fibers, which are present in many biological tissues cause anisotropic response. Although it is known that biological tissues exhibit hysteretic behavior in cyclic loading, it is most common to ignore this dissipative response and model tissues as anisotropic hyperelastic materials.

Some authors have proposed models for the dissipative response of tissues. Phenomenological models include the work of Rubin et al. (1998) and Rubin and Bodner (2002), which present constitutive equations for modeling the dissipative response of soft tissues. Also, Nasseri et al. (2002) use a multi-component Maxwell model to characterize viscoelastic properties of pig skin in shear. Bischoff et al. (2004) proposed a 15-parameter Langevin model to characterize soft tissue viscoelasticity using a three-element rheological network model introduced by Bergstrom and Boyce (1998). Bischoff (2006) and Flynn et al. (2011b) model the rheology of biological tissues using the concept of quasi-linear viscoelasticity (QLV) introduced by Fung (1993). However, soft tissues exhibit nonlinear viscoelasticity, in general, and the assumptions of the QLV model do not hold in all cases (Einat and Lanir, 2009). Another approach is to use structurally based models, which characterize the collective response of fiber bundles. Examples of these structural models include the works of Lanir (1979) and Lokshin and Lanir (2009), which use the quasi-linear viscoelastic response of fibers to capture the rheology of tissues. In particular, Lokshin and

* Corresponding author. Tel.: +64 7 834 8800x3113.
E-mail addresses: cormac.flynn@wintec.ac.nz (C. Flynn), mbrubin@tx.technion.ac.il (M.B. Rubin).

0167-6636/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.mechmat.2013.07.009

Lanir (2009) compared their model predictions with a large set of experimental theoretical data. Ehret et al. (2009, 2010) have developed a three-dimensional anisotropic generalization of the standard three-parameter solid to develop structural models for soft biological and collagenous tissues. Also, mention is made of a recent review of the constitutive modeling of soft biological tissues that can be found in Ehret (2011). In particular, Chapter 6 in Ehret (2011) describes dissipative effects, which are modeled using evolution equations for generalized structural tensors.

One objective of the present work is to introduce simple constitutive equations, which can model dissipative response as well as anisotropic response due to a discrete set of fibers. This model has features of both the phenomenological and structural models described above. Specifically, the model can be used to quantify the potential influence of inelastic dissipation on cyclic loadings that occur in essential activities like heart pumping and breathing.

Here, it is assumed that the biological tissue is a composite of elastic fibers embedded in a dissipative matrix. The strain energy function for the biological tissue depends on the total dilation J to control volume changes, on a modified generalized orthotropic invariant γ to control anisotropic response to distortional deformation, and on a measure α of elastic distortional deformation of the dissipative matrix to predict hysteretic behavior. The generalized invariant is based on six discrete fiber bundles as described in Flynn and Rubin (2012) and is modified to limit attention to distortional deformations. The dissipative matrix is modeled using the elastically isotropic constitutive equations developed in Hollenstein et al. (2013), which model a smooth inelastic transition. Specifically, here the hardening variable is modified to include explicit dependence on the total strain. An important feature of the constitutive equation is that the evolution equation for the inelastic response can be integrated numerically using a strongly objective algorithm that needs no iteration. The resulting tissue model exhibits anisotropic and dissipative response, which is nearly rate-independent for high enough loading rates. It also exhibits rate-dependent stress relaxation, and a simplified form of preconditioning. Since the strain energy function for distortion of the tissue depends on both an elastic deformation measure of distortion of the dissipative matrix and on a total distortional deformation measure of the fiber bundles in a coupled form, inelasticity is exhibited by both the matrix and the fibers. Also, the entire constitutive model is properly invariant under Superposed Rigid Body Motions (SRBM). In addition, the equations can easily be programmed into finite element computer codes to model complicated tissue response.

An outline of this paper is as follows. Section 2.1 presents the basic equations of the model and Section 2.2 presents specific simple forms for the functions. Section 2.3 discusses the numerical integration scheme and Section 3.1 presents examples showing the influence of the material constants. Section 3.2 shows comparisons of the predictions of the model with experimental data and Section 4 presents discussion and conclusions.

2. Material and methods

2.1. Basic equations

Following the work in Flynn and Rubin (2012), Elata and Rubin (1994, 1995) and Flynn et al. (2011a) use is made of the unit vectors $\mathbf{N}_i$ (i = 1,2,. . .,6), which characterize the directions of opposing vertices of a regular icosahedron given by

$$\mathbf{N}_1 = \frac{2}{\sqrt{5}}\mathbf{e}_1 + \frac{1}{\sqrt{5}}\mathbf{e}_3, \quad \mathbf{N}_2 = \frac{1}{2}\left(1 - \frac{1}{\sqrt{5}}\right)\mathbf{e}_1 + \sqrt{\frac{1}{2}\left(1 + \frac{1}{\sqrt{5}}\right)}\ \mathbf{e}_2 + \frac{1}{\sqrt{5}}\mathbf{e}_3,$$

$$\mathbf{N}_3 = -\frac{1}{2}\left(1 + \frac{1}{\sqrt{5}}\right)\mathbf{e}_1 + \sqrt{\frac{1}{2}\left(1 - \frac{1}{\sqrt{5}}\right)}\ \mathbf{e}_2 + \frac{1}{\sqrt{5}}\mathbf{e}_3,$$

$$\mathbf{N}_4 = -\frac{1}{2}\left(1 + \frac{1}{\sqrt{5}}\right)\mathbf{e}_1 - \sqrt{\frac{1}{2}\left(1 - \frac{1}{\sqrt{5}}\right)}\ \mathbf{e}_2 + \frac{1}{\sqrt{5}}\mathbf{e}_3,$$

$$N_5 = \frac{1}{2}\left(1 - \frac{1}{\sqrt{5}}\right)\mathbf{e}_1 + \sqrt{\frac{1}{2}\left(1 + \frac{1}{\sqrt{5}}\right)}\ \mathbf{e}_2 + \frac{1}{\sqrt{5}}\mathbf{e}_3, \quad \mathbf{N}_6 = \mathbf{e}_3, \tag{1}$$

where $\mathbf{e}_i$ (i = 1,2,3) are fixed rectangular Cartesian base vectors. Moreover, let $\mathbf{B}_i$ be six constant symmetric second order tensors with the properties

$$\mathbf{B}_i = \mathbf{N}_i \otimes \mathbf{N}_i \quad (\text{no sum on } i = 1,2,\ldots,6), \quad \sum_{i=1}^{6}\mathbf{B}_i = 2\mathbf{I},$$

$$\mathbf{B}_i \cdot \mathbf{B}_j = 1 \quad \text{for } i = j, \quad \mathbf{B}_i \cdot \mathbf{B}_j = \frac{1}{5} \quad \text{for } i \neq j$$
$$(i,j = 1,2,\ldots\ldots,6), \tag{2}$$

where $\otimes$ is the tensor product operator, $\mathbf{I}$ is the second order unit tensor, $\mathbf{a} \cdot \mathbf{b}$ denotes the scalar product between two vectors and $\mathbf{A} \cdot \mathbf{B} = \mathrm{tr}(\mathbf{AB}^T)$ denotes the scalar product between two second order tensor $\{\mathbf{A},\mathbf{B}\}$. Furthermore, let $\mathbf{W}$ be a constant symmetric structural tensor (Flynn and Rubin, 2012) defined by

$$\mathbf{W} = \sum_{i=1}^{6} w_i\mathbf{B}_i, \quad w_i \geqslant 0, \quad \mathbf{W} \cdot \mathbf{I} = \sum_{i=1}^{6} w_i = 1, \tag{3}$$

where w_i are non-negative constant weights. These conditions on the weights ensure that $\mathbf{W}$ is a positive semi-definite tensor, since for an arbitrary unit vector $\mathbf{n}$

$$\mathbf{W} \cdot (\mathbf{n} \otimes \mathbf{n}) = \sum_{i=1}^{6} w_i(\mathbf{N}_i \cdot \mathbf{n})^2 \geqslant 0. \tag{4}$$

Furthermore, it is noted that in Flynn and Rubin (2012) it was shown that the weights w_i and the fiber bundle orientations $\mathbf{B}_i$ can be used to define a distribution function of fiber bundle orientations which can be compared with histological evidence.

In order to characterize the anisotropic response of the tissue, it is recalled that the deformation gradient $\mathbf{F}$ from a