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Wave propagations in exponentially graded transversely isotropic half-space with potential function method

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ABSTRACT

Time-harmonic response of a vertically graded transversely isotropic, linearly elastic half-space is analytically determined by introducing a new set of potential functions. The potential functions are set in such a way that the governing equations be simple and with physical meaning as well. In addition, the potential functions introduced in this paper are degenerated to a complete set of potential functions used frequently for wave propagations in homogeneous transversely isotropic media. Utilizing Fourier series and Hankel integral transforms, the governing equations for the potential functions are solved, after which the displacements and stresses are presented in the form of line integrals. Both the displacements and stresses determined here are collapsed on the solution previously reported for the constant profile transversely isotropic material. Because of complicated integrand functions, the integrals are evaluated numerically and presented graphically, where the effect of degree of change of material properties plays a major role, which may be recognized easily.

1. Introduction

Wave propagation in solids induced by external loading has long been the subject of numerous investigations. Traditionally, when one studies the phenomenon of wave propagation in solids, the media are often assumed to be homogeneous, isotropic, and linearly elastic continua (see for example Lamb, 1904; Ewing et al., 1957; Achenbach, 1973; Aki and Richards, 1980; Apsel et al., 1979; Apsel and Luco, 1983; Miklowitz, 1978; Pak, 1987; Pak and Guzina, 2002). However, there are many natural soils deposited through a geologic process of sedimentation over a period of time, such as flocculated clays, silts and sands, or rocks, foliated metamorphic, stratified sedimentary, regularly jointed rocks, where determining their displacements and stresses needs to take into account the anisotropy (Wang et al., 2006). In addition, from the practical engineering point of view, many anisotropic soils are often modeled as transversely isotropic media. Moreover, the mechanical response of anisotropic materials with spatial gradients in composition is of considerable interest in soil/rock mechanics (Suresh, 2001). The effects of deposition, overburden, desiccation, etc., can lead geological media, which exhibit both inhomogeneity and anisotropic deformability characteristics (Wang et al., 2006). Thus, the desire to understand the wave propagation in an anisotropic medium has risen substantially from the recognition that the anisotropy of materials is the norm rather than an exception. However, because of mathematical difficulty associated with these media, theoretical understanding of the phenomena has not received much attention. As demonstrated by Stoneley (1949), the presence of transverse isotropy can result in significant differences in wave propagation compared with the case of isotropic materials. Likewise, Synge (1957) studied the propagation of Rayleigh waves in a transversely isotropic medium and found that they will propagate only if the free surface of the material is parallel or perpendicular to the material axis of symmetry.

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Notation

$c_{0\,ij}$	lasticity constants at z=0
s	depth of point load
C_d	dilatational wave speed
C_s	equivoluminal wave speed
E	Young's moduli in the plane of transverse isotropy
$E^{'}$	Young's moduli in the direction normal to the plane of transverse isotropy
G	shear modulus in the plane normal to the axis of symmetry
$G^{'}$	shear modulus in planes normal to the plane of transverse isotropy
J_m	Bessel function of the first kind and mth order
$P(r, \theta)$	time-harmonic surface force component in r-direction
$Q(r, \theta)$	time-harmonic surface force component in θ-direction
$R(r, \theta)$	time-harmonic surface force component in z-direction
r	radial coordinate
t	time variable
u_i	displacement component in i direction ($i = r, \theta, z$)
z	vertical coordinate
$\delta(r)$	Dirac-delta function
ε_{ij} ($i, i = r, \theta, z$)	strain components
β	non-homogeneity parameter
θ	angular coordinate
$\lambda_1, \lambda_2, \lambda_3$	radicals appearing in general solutions
ξ	Hankel's parameter
ρ_0	material density
σ_{ij} ($i, j = r, \theta, z$)	stress tensor
ω_0	non-dimensional frequency
ω	angular frequency

The method of potential function is a powerful tool to study three-dimensional wave propagations in solids, analytically. So far, a few sets of complete potential functions have been introduced to uncouple the system of equations of equilibrium, equations of motion or equations of motion coupled with heat transfer equation in transversely isotropic materials (Lekhnitskii, 1940; Hu, 1953; Nowacki, 1954; Elliott, 1948; Lodge, 1955; Kellogg, 1953; Eskandari-Ghadi, 2005; Eskandari-Ghadi et al., 2009; Eskandari-Ghadi et al., 2010). In addition, there are some researches containing the response of transversely isotropic materials as Green's functions or to some loads other than point load in either elastostatic or elastodinamic cases and to forced vibrations of some rigid plates, where the method of potential functions have been utilized (Pan and Chou, 1976; Pan and Chou, 1979; Rahimian et al., 2007; Eskandari-Ghadi et al., 2008) Eskandari-Ghadi et al., 2011; Eskandari-Ghadi et al., 2012; (Eskandari-Ghadi and Ardeshir-Behrestaghi, 2010).

On the other hand, because of sedimentary process and confining pressure, the properties of soil vary in depth. Thus, assuming constant depth-profile properties for soil deposits and some types of rocks may result in a rather poor approximation compare to the real conditions. Thus, studying wave propagation in inhomogeneous media is a need in soil-structure-interaction point of view. Functionally graded materials (FGMs) having the desired variation of material properties in spatial directions are widely used in different applications such as aerospace and automobile industries (Eskandari and Shodja, 2010). Some attempts have been made for this class of engineering problems in which several kind of variations such as linear variation (Gibson Soil) and exponential variation have been considered (Wang et al., 2006). However, in between different kind of in-homogeneity, the exponential variation of the elasticity tensor is widely used for FGMs in the engineering literature (Martin et al., 2002).

On the other hand, semi analytical/numerical method such as boundary element method may be useful for deep analysis of this media (Pan and Han, 2004). Displacement- and stress-Greens functions are prerequisite for using BEM in analyzing FGM media. There are a few researches, where one may find the Green's functions for either isotropic or anisotropic FGMs in the elastostatic cases (Pak and Guzina, 1995; Martin et al., 2002; Wang et al., 2003; Chan et al., 2004; Wang et al., 2006; Kashtalyan and Rushchitsky, 2009; Eskandari and Shodja, 2010). For example, (Pak and Guzina, 1995), by extending a previously reported potential functions for homogeneous isotropic material, analyzed a heterogeneous isotropic media in frequency domain, and (Kashtalyan and Rushchitsky, 2009), with the aid of the method of displacement potential functions, have solved equilibrium equation in non-homogenous media in the case of elastostatics.

In this paper, a vertically graded transversely isotropic half-space in the form of exponential function is considered as the domain of interest of this study. The half-space is considered in such a way that its material axis of symmetry to be depth-wise at any point. An arbitrary time-harmonic patch load is assumed to be applied at an arbitrary depth from the surface of the half-space. By extending Eskandari-Ghadi's potential functions (Eskandari-Ghadi, 2005) for this class of functionally graded transversely isotropic materials, a new set of potential functions containing two scalar potential functions is introduced to uncouple the equations of motion. The governing equations for the potential functions are in the form of a second order and a forth order non-constant coefficients partial differential equation, whose solutions are determined by virtue of Fourier series and Hankel integral transforms. Although the ordinary differential equations governing one of the potential functions is not with constant coefficients, its solution is given in closed form. The displacements and stresses are introduced in