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A new theoretical model of the quasistatic single-fiber pullout problem: Analysis of stress field

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ABSTRACT

A new theoretical model is developed in order to predict the stress transfer during the quasistatic single-fibre pullout process. The theoretical approach retains all relevant stress and strain components, and satisfies exactly the interfacial continuity conditions and all the stress boundary conditions. For both matrix and fibre, the equilibrium equations along radial direction are satisfied strictly, while the equilibrium equations along axial direction are satisfied in the integral forms. Three normal stress–strain relationships are strictly satisfied, while the radial displacement gradient with respect to the axial direction is neglected for shear stress–strain relationship. The general solutions of the axial and radial displacements in both fibre and matrix are obtained in explicit forms. In the debonded region, a modified Coulomb's friction law, in which the frictional coefficient is a decreasing function of pullout rate, is applied to determine the interfacial frictional stress. The new analytical approach allows performing more detail theoretical analysis on the stress transfer between fibre and matrix, and distributions of stress, strain and displacement in fibre and matrix. Numerical results of the stress distributions, in both fully bonded region and fully debonded region, are presented for a typical glass/epoxy composite system with different fibre volume fraction and model length. In fully bonded region, the theoretical results from present model are more accurate compared with those from Lame solution, and agree well with the results from finite element model. In fully debonded region, present model can predict the initial pullout stress under different geometrical conditions and static friction coefficient, also can predict more reasonable stress distribution than Lame solution.

1. Introduction

It is well recognized that the interface between fibre and matrix plays an important role in the mechanical properties of fibre-reinforced composite (Kim and Mai, 1998). Interface debonding and sliding with friction are two main contributors to the dissipated energy. During the past several decades, extensive experimental methods (pull-out, push-out, fibre fragmentation and so on) have been developed and exploited to study the influence of interface on overall response of composite. The single-fibre pullout test attracts increasing attention amongst these test methods because of its versatility and simplicity. The load–displacement curve from the pullout test can be applied in two fields: to extract the interfacial properties (Marshall, 1992; Marshall et al., 1995; Hsueh, 1993), such as interfacial strength and friction coefficient, and to study the fracture behavior of a fibre-bridge crack in a fibre-reinforced composite (Thouless and Evans, 1988; Evans, 1989; Thouless et al., 1989).

A number of models have been developed to understand how and how much the interface properties affect the mechanical performance and fracture behavior of the fibre composite. To the best knowledge of the author, the earliest theoretical work can be traced back to 1952

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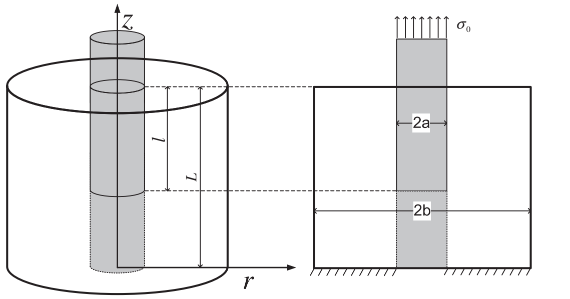

Fig. 1. Schematic diagram for fibre pullout of the coaxial single-fibre/matrix cylinder.

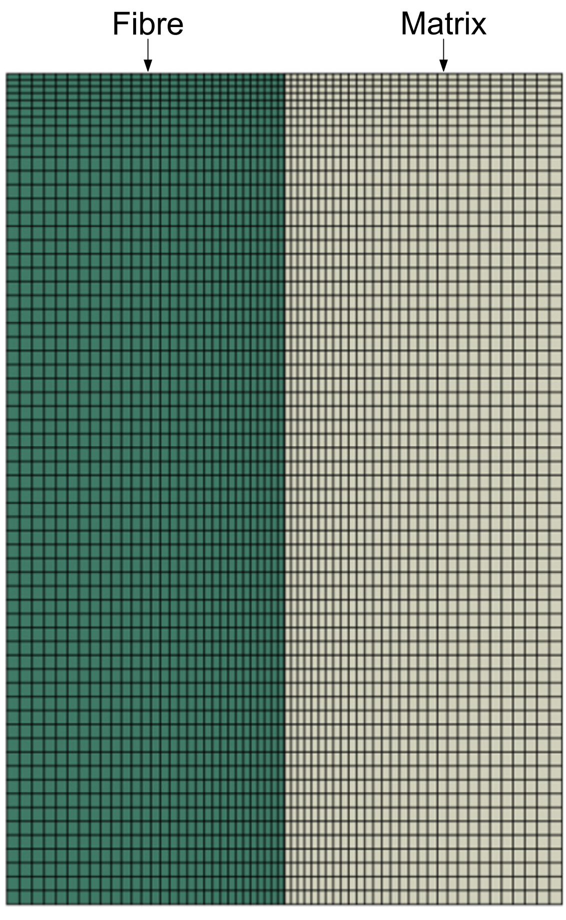

Fig. 2. FE-model of single-fibre pullout.

(Cox, 1952). The limitations existing in the previous analysis can be classified into following five groups:

- A plane strain condition is assumed in order to relate the tangential stress to the radial stress at the interface (Cox, 1952; Rosen, 1964; Gao et al., 1988; Hutchinson and Jensen, 1990; Hsueh, 1990a,b; Zhou et al., 1992a, 1992b; Zhou and Mai, 1993; Kim et al., 1994; Liu et al., 1994; Zhang et al., 1999).
- One or two of the equilibrium equations are disregarded (Cox, 1952; Rosen, 1964; Gao et al., 1988; Hutchinson and Jensen, 1990; Zhou et al.,1992a,b; Zhou and Mai, 1993; Kim et al., 1994; Liu et al., 1994; Zhang et al., 1999).
- The axial stress either in fiber or matrix, or in both, is independent on the radial coordinate (Cox, 1952; Rosen, 1964; Gao et al., 1988; Mccartney, 1989; Hsueh, 1990a,b; Hutchinson and Jensen, 1990).
- Poisson effect is partially or fully neglected (Cox, 1952; Rosen, 1964; Hsueh, 1990a,b, 1992, 1995).
- Shear stress in matrix satisfies the free boundary conditions only in an approximate sense. (Cox, 1952; Rosen, 1964; Hsueh, 1990a,b).

Nairn (1992, 1997) developed a model for the single-fibre fragmentation test through variational mechanics. The model assumes that axial normal stresses depend only on the axial locations, satisfies most boundary conditions, and obeys the compatibility approximately. His model is not applicable for fibre pullout test, because the stress functions are based on the specific boundary conditions for the fibre fragmentation test. Meanwhile, the final solution can only be obtained numerically because of the introduce of the rather complicated mathematical derivations.

Furthermore, one important assumption of the existed theoretical models is that the interfacial shear stress is governed by a classical Coulomb friction law, which can be expressed as

$$\tau_i = -\mu\sigma_{ri} \tag{1}$$

where, τ_i is the interfacial shear stress, σ_{ri} is the interfacial radial stress, and μ is a constant indicating the friction coefficient. The negative sign in Eq. (1) indicates that the friction is caused by the compressive radial stress, and the friction is zero if the radial interfacial stress is in tension state ($\sigma_{ri} > 0$).

Previous study shows, during the single-fibre pullout process, the maximum static friction between two surfaces is larger than the kinetic friction, and friction exhibits velocity-weakening phenomenon (Rice and Ruina, 1983; Tsai and Kim, 1996; Hashiguchi et al., 2005). In the quasi-static state, a general form of the frictional coefficient can be expressed as

$$\mu = \mu_k + (\mu_s - \mu_k)e^{-\left(\frac{v_t}{v_0}\right)^p} \tag{2}$$

In which, v_0 is a reference speed, v_t is the pullout speed, p is the damping ratio, μ_s is the stick–slip frictional coefficient ($v_t = 0$), and μ_k is limiting dynamic frictional coefficient ($v_t \to \infty$). Generally, $p > 0$ and $\mu_s > \mu_k$, which indicates that the frictional coefficient is a decreasing function of the velocity imposed at the fibre loading end in stick–slip state.

The purpose of the present work is to overcome all above mentioned limitations and develop a new theoretical model for a quasistatic single-fiber pullout process. The theoretical model retains all the relevant stress and strain components, and satisfies exactly stress–strain relationships, and the interface conditions and other boundary conditions involving stresses. The interfacial shear stress