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Fields produced by three-dimensional dislocation loops in anisotropic magneto-electro-elastic materials

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ABSTRACT

In this article, we analyze the coupled elastic, electric and magnetic fields produced by an arbitrary three-dimensional dislocation loop in general anisotropic magneto-electro-elastic materials. We first extend the anisotropic elastic formulae of dislocations to the corresponding magneto-electro-elastic material system, including a general line-integration solution and the solution of a straight-line segment of dislocation. We then develop a new line-integral solution for the extended displacement field as well as the extended stress field. Furthermore, we derive analytical expressions for some useful parametric dislocation curves, such as the elliptic arc and straight line. Our solutions contain the piezoelectric, piezomagnetic, and purely anisotropic elastic solutions as special cases. As numerical examples, the fields produced by elliptic, hexagonal and cardioid shape dislocation loops in both piezoelectric crystals and magneto-electro-elastic materials are calculated. The efficiency and accuracy of different integral solutions of dislocation loops are compared and discussed. More important, the coupling magneto-electro-elastic effect is illustrated. It is shown that, due to the coupling among the elastic, electric and magnetic fields, an elastic dislocation, an electric potential discontinuity, or a magnetic potential discontinuity can induce all the elastic, electric and magnetic fields and that the coupling effect could be very strong near the dislocation loop line.

1. Introduction

The piezoelectric material, which possesses the coupling effect between mechanical and electric fields, is now being widely applied to different engineering technologies. This stimulates various theoretical studies on such coupling materials. One of the studies is on the dislocations and their movements since they play an important role in the physical behaviors of materials. While fracture mechanics problems in piezoelectric materials have been well investigated for both two-dimensional and three-dimensional problems (e.g., Suo et al., 1992; Zhang et al., 2002), the corresponding dislocation problems were mostly investigated for two-dimensional domains only (e.g., Pak, 1990; Liu et al., 1999; Chen et al., 2004; Wang and Sudak, 2007) where the dislocations were taken to be infinite straight line. In reality, however, dislocations usually form three-dimensional loops which are more difficult to analyze. Although Minagawa and Shintani (1985), Minagawa (2003) studied the stress and electric fields produced by dislocation loops, the procedure and solution were complicated and only the elastic displacement dislocation was considered. Nowacki and Alshits (2007) extended the dislocation-field expression to piezoelectricity but no numerical example was given. Dislocations in piezoelectric and magneto-electro-elastic (MEE) materials could show some interesting features and deserve further investigation. For instance, due to the coupling between

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the mechanical and electric fields, a moving dislocation in piezoelectric crystal could induce certain interesting coupling features (Soh et al., 2005). Further due to the coupling, the dislocation mobility (Li and Gupta, 2004) and energetics of a partial dislocation (Belabbas et al., 2006) could be different than those in the purely elastic domain. Other important and potential applications of the dislocation solution in piezoelectric materials include the induced polarization feature in such materials (Shi et al., 1999). Since most technically important materials would be also ferroelectric, the dislocation solution could be further applied to study the dislocation-induced polarization variation and distributions (Zheng et al., 2006), their effect on ferroelectric phase stability, domain morphology (Hu et al., 2003) and possible degradation of ferroelectric properties (Alpay et al., 2004). Composites made of piezoelectric/piezomagnetic materials exhibit magnetoelectric coupling effect that is not present in the single-phase piezoelectric or piezomagnetic material. In the past decade, much attention has been paid to predict the effective properties of MEE composites according to the theories of micromechanics. But for dislocation problems in MEE, relatively little work has been done. Until now, only one-dimensional dislocations in such coupling materials were studied (Hao and Liu, 2006; Ma and Lee, 2007; Lee and Ma, 2010).

Motivated by the important applications of the MEE material and the potential influence of dislocations on such a material, we derive, in this paper, the extended stress fields induced by the extended dislocation loops by utilizing the extended Green's functions and their derivatives in MEE materials. We present three forms of the solutions: a line integral form for smooth dislocation loops which can be evaluated by a standard numerical integration method, an analytical expression which is for the loops made of piecewise straight lines, and the analytical solution for some parametric curve loops, such as elliptic arcs. Numerical examples are presented for elliptic, hexagonal and cardioid dislocation loops in piezoelectric materials GaAs and AlN and in MEE composites made of $BaTiO_3$–$CoFe_2O_4$. Our results show clearly the important coupling features among mechanical, electric and magnetic fields.

2. Basic equations

With the extended notation (Barnett and Lothe, 1975), the equilibrium equations (including the electric and magnetic balance equations) and the constitutive relations for the coupled MEE media can be expressed as (Pan, 2002):

$$\sigma_{iJ,i} + f_J = 0, \quad \sigma_{iJ} = C_{iJKl}\gamma_{Kl} \tag{1}$$

The summation over repeated lowercase (uppercase) subscripts is from 1 to 3 (1–5), and a subscript comma denotes the partial differentiation with respect to the coordinates. The extended displacement, body force, strain and stresses are defined as

$$u_I = \begin{cases} u_i, & I = i = 1,2,3 \\ \phi, & I = 4 \\ \psi, & I = 5 \end{cases} \tag{2a}$$

$$f_J = \begin{cases} f_j, & J = j = 1,2,3 \\ -f_e, & J = 4 \\ -f_m, & J = 5 \end{cases} \tag{2b}$$

$$\gamma_{Ij} = \begin{cases} \gamma_{ij} = 0.5(u_{i,j} + u_{j,i}), & I = i = 1,2,3 \\ -E_j = \phi_{,j}, & I = 4 \\ -H_j = \psi_{,j}, & I = 5 \end{cases} \tag{2c}$$

$$\sigma_{iJ} = \begin{cases} \sigma_{ij}, & J = j = 1,2,3 \\ D_i, & J = 4 \\ B_i, & J = 5 \end{cases} \tag{2d}$$

and the extended elastic coefficient matrix has the following components

$$C_{iJKl} = \begin{cases} C_{ijkl}, & J,K = j,k = 1,2,3 \\ e_{lij}, & J = j = 1,2,3;\ K = 4 \\ e_{ikl}, & J = 4;\ K = k = 1,2,3 \\ q_{lij}, & J = j = 1,2,3;\ K = 5 \\ q_{ikl}, & J = 5;\ K = k = 1,2,3 \\ -\alpha_{il}, & J = 4;\ K = 5 \text{ or } J = 5; K = 4 \\ -\varepsilon_{il}, & J = K = 4 \\ -\mu_{il}, & J = K = 5 \end{cases} \tag{3}$$

In Eqs. (1), (2a), (2b), (2c), (2d), (3), u_i, ϕ and ψ are the elastic displacement, electric potential and magnetic potential; f_i, f_e and f_m are the body force, electric charge and electric current; γ_{ij}, E_i and H_i are the strain, electric field and magnetic field; σ_{ij}, D_i and B_i are the stress, electric displacement and magnetic induction, respectively; C_{ijlm}, ε_{ij}, and μ_{ij} are the elastic, dielectric and magnetic permeability tensors, e_{ijk}, q_{ijk} and α_{ij} are the piezoelectric, piezomagnetic, and magnetoelectric coefficients, respectively. The material constants satisfy the following symmetry relations:

$$\begin{aligned} &C_{ijkl} = C_{ijlk} = C_{jikl} = C_{klij}; \varepsilon_{ij} = \varepsilon_{ji}; \mu_{ij} = \mu_{ji} \\ &e_{kij} = e_{kji}; q_{kij} = q_{kji}; \alpha_{ij} = \alpha_{ji} \end{aligned} \tag{4}$$

It is noted that we assumed that the magnetoelectric coefficient matrix α_{ij} is symmetric and that we have

$$C_{iJKl} = C_{lKJi} \quad \text{but} \quad C_{iJKl} \neq C_{iJlK} \neq C_{JiKl} \neq C_{KliJ} \tag{5}$$

The extended Green's functions (5 × 5 tensor) $G_{KM}(y;\boldsymbol{x})$ are defined as the extended displacement component $u_K(\boldsymbol{x})$ at a field point $\boldsymbol{x}$ due to an extended unit point force in M-direction at the source point $\boldsymbol{y}$. They satisfy the equilibrium equation

$$[C_{iJKl}(\boldsymbol{x})G_{KM,x_l}(\boldsymbol{y};\boldsymbol{x})]_{,x_i} + \delta_{JM}\delta(\boldsymbol{y};\boldsymbol{x}) = 0 \tag{6}$$

with $f_{,x_i} = \partial f/\partial x_i$, δ_{JM} being the fifth-rank Kronecker delta, and $\delta(\boldsymbol{y};\boldsymbol{x})$ the Dirac-delta function which is zero everywhere except at point $\boldsymbol{x} = \boldsymbol{y}$. The solutions of the extended Green's functions and their derivatives are given in Appendix A.

Consider a region V in the 3D space which is bounded by the surface S. If $\boldsymbol{x}$ is inside the region, then multiplying Eq. (6) by u_J and integrating through the region, we have

$$u_M(\boldsymbol{y}) = -\int_V [C_{iJKl}(\boldsymbol{x})G_{KM,x_l}(\boldsymbol{y};\boldsymbol{x})]_{,x_i} u_J(\boldsymbol{x})dV(\boldsymbol{x}) \tag{7}$$