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Relationship between fracture and friction for brittle rocks

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Abstract

We fit a recently proposed failure criterion to experimental data for fracture and friction strength of various rocks. The criterion rests on the Cardano condition for the existence of three real-valued eigenvalues of the characteristic equation of the stress tensor and specifically addresses the non-linear dependence of fracture and friction on increasing mean stress. The approach provides a theoretical link between macroscopic fracture of a continuum and sliding friction on a pre-existing interface. The criterion rests on three fit parameters whose physical meaning is investigated by fitting previously published data of rocks characterized by differences in composition, porosity, and grain size. In all investigated cases the friction characteristics predicted from the fit of fracture strength data excellently fit independent experimental friction data. Of the three fit parameters, one appears to be determined by the stress concentrations associated with pores and an intrinsic, composition-dependent strength parameter of the constituents, such as mineral-hardness. The second parameter scales inversely with grain size and represents the tensile strength of the polycrystalline aggregates. Mostly owing to the lack of explicit tensile strength or extension data, the third parameter quantifying the pressure-dependence at very low mean stresses remains poorly constrained. It appears affected by composition or the geometric distribution of phases. Fitting macroscopic failure accompanied by pressure-insensitive, crystal plastic deformation mechanisms, such as mechanical twinning in marbles, requires much lower values of the third parameter than purely brittle deformation.

Keywords: Rock failure; Experimental rock deformation; Failure criterion; Brittle fracture; Friction; True triaxial

1. Introduction

Predicting and estimating the load-bearing capacity of rocks is crucial for problems on the scales of geo-engineering constructions to plate tectonics. Strength is an often loosely used term denoting a stress or strain limit beyond which further loading leads to failure, i.e., inelastic deformation. Here, we use the term failure as inclusive for macroscopic fracture, cataclastic flow, and frictional sliding on a pre-existing interface. In order to determine characteristic measures of strength, rock samples are subjected to uniaxial tensile loading ($\sigma_1 > 0$, $\sigma_2 = \sigma_3 = 0$), conventional triaxial extension ($\sigma_1 = \sigma_2 \equiv \sigma_c$) and compression ($\sigma_2 = \sigma_3 \equiv \sigma_c$), or true triaxial loading ($\sigma_1 \neq \sigma_2 \neq \sigma_3$). We use the engineering convention that compressive stresses are

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negative and index principle stresses, σ_i, in the order of their absolute size. The vast majority of tests is conducted in conventional triaxial loading of cylindrical samples where two principal stresses are determined by the pressure of a confining fluid (confining pressure $P_c > 0$) and are thus identical, only the principal stress in the direction of the cylinder axis differs in magnitude. Yet, the mechanical behavior of rocks at true triaxial stresses was studied, too (e.g., Mogi, 1971; Celle and Cheatham, 1976; Chang and Haimson, 2000; Haimson and Chang, 2000).

Up to moderate mean stress levels, failure of intact samples is characterized by sudden loss of load-bearing capacity and the formation of a single or few fracture planes dissecting the sample. Such brittle failure is believed to result from coalescence of microcracks as indicated by quasi online monitoring of volumetric strain (e.g., Brace et al., 1966), permeability, (e.g., Renner et al., 2000), acoustic velocity, (e.g., Schubnel et al., 2003), and activity of acoustic emissions, (e.g., Lockner et al., 1992) during deformation. The angle between failure planes and the maximum compressive stress generally increases with increasing mean stress. Failure strength is affected by the intermediate stress; an increase in mean stress is associated with an increase in strength. The dip between the failure plane and the maximum principal stress decreases with increasing intermediate stress.

Frictional characteristics of rock interfaces are determined in direct shear experiments (biaxial: $\sigma_1 \neq \sigma_2$, $\sigma_3 = 0$) but also in conventional triaxial set-ups. Shear planes are either prepared by cutting and polishing pieces of rocks, sampling natural joints, or experimentally inducing localized failure. Results are presented as friction criteria. Amonton's law relates the sliding frictional strength or frictional resistance denoted by the maximum shear stress τ_f to the normal stress σ_n acting on a plane by

$$\tau_f = \mu_s \sigma_n, \tag{1}$$

introducing the coefficient of sliding friction μ_s. From friction experiments two trends emerged, now known as Byerlee's rule in geo-science (Byerlee, 1978): (1) the coefficient of sliding friction decreases with increasing normal stress; (2) frictional resistance is rather insensitive to the composition of rocks as long as certain minerals such as sheet silicates or clays are excluded. From tests on a variety of rocks, Byerlee derived the following empirical bi-linear relation (Lockner, 1995):

$$\begin{aligned} &\tau_f = 0.60\sigma_n - 50\ \text{MPa} \\ &\quad \text{for } -200\ \text{MPa} \geqslant \sigma_n > -1700\ \text{MPa}, \\ &\tau_f = 0.85\sigma_n \quad \text{for } 0 \geqslant \sigma_n > -200\ \text{MPa}. \end{aligned} \tag{2}$$

Note, according to these rules friction is not associated with a physical cohesion. However, a finite shear stress for vanishing normal stress apparently occurs when the relation for high normal stresses is extrapolated outside of its range of validity. To these rules of thumb the general finding may be added that macroscopic sliding friction coefficients usually exceed values for individual mineral interfaces.

Probably in mere analogy to the description of sliding friction, Coulomb proposed the first fracture criterion in 1776 (Timoshenko, 1953)

$$\tau_s = S_0 + \mu_i \sigma_n. \tag{3}$$

The slope μ_i of the linear relation between shear stress at failure and normal stress with respect to the emerging fault plane expresses the notion that failure is related to overcoming internal friction. In addition, an intrinsic resistance, S_0, has to be overcome, i.e., cohesion was rather considered a property of a continuum than part of a friction law. The slope of the failure criterion, often referred to as the coefficient of internal friction (Savage et al., 1996), significantly decreases with increasing mean stress (Byerlee, 1967; Scott and Nielsen, 1991a; Shimada et al., 1983) and eventually becomes less then the coefficient of friction determined from the resistance against sliding on pre-existing fault planes. The intersection of fracture and friction criterion indicates that generating a new fracture plane and activating an existent fault require identical deformation energy (Byerlee, 1967).

Frictional and fracture strength differ by describing a property of an interface and a continuum, respectively. In compression, macroscopic failure likely results from the coalescence of interacting micro-flaws. In contrast, failure in tension may be caused by the instability of a single pre-existing micro-flaw as initially considered by Griffith (1924). The transition from tensile to compressive failure mode is controlled by hydrostatic pressure. Guided by the notion of microcrack-activity, significant theoretical effort was spent on linking the coefficient of internal friction and the coefficient of sliding friction of individual mineral contacts (e.g., Ashby and Sammis, 1990). Yet, relating macroscopic fracture and friction strength remained enigmatic because fracture strength varies over orders of