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Singularity analysis of a planar robotic manipulator: Application to an XY-Theta platform

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ABSTRACT

This paper deals with the singularity analysis of a planar robotic manipulator and its application for an XY-Theta platform. This platform has a patented kinematics designed to keep the final position error below 2 μm in its workspace. But as the high precision performances are due to the proximity of singularities, some drawbacks such as high joint velocities and torques may appear when the trajectory is too close to singularities. Therefore, the main objective of this paper is to identify the singularity loci. Usually, when a non-redundant robot operates in a 3D space, the singularity locus is a surface. Here, one contribution is the identification of an helicoidal line for the singularity locus within the workspace. The specific control problems linked to this singularity locus are then described in details. Another contribution concerns the identification of the singularity loci taking into account the robot redundancy and the analysis of the induced control problems linked to the crossing of singularity surfaces. Finally, the manipulability index is computed. It estimates the distance between the current position and the singularity configuration and will be used in the future to keep the position away from singularity.

1. Introduction

It is highly recommended to identify the singularities of a manipulator during the design stage, in order to take into account their location for the control of the robot. Indeed, near singular configurations, the robot control algorithms may lead to large joint velocities or encounter instantaneous loss of dof (degree of freedom) [1]. This singularity analysis has been performed by many researchers on several different robots [2,3].

A robot has n dof in the operational space described by $\boldsymbol{x} = (x_1, \ldots, x_n)$ and m degrees of mobility in the actuator space described by $\boldsymbol{q} = (q_1, \ldots q_m)$. For serial robots, actuator and operational coordinates are linked by the forward kinematic function $\boldsymbol{x} = f(\boldsymbol{q})$. The Jacobian matrix appears after differentiating this relation: $d\boldsymbol{x} = \boldsymbol{J}d\boldsymbol{q}$. Mathematically, considering the case of non-redundant serial robots, a loss of one or more dof is characterized by a drop in the rank of the Jacobian, implying $det(\boldsymbol{J}) = 0$. If the serial robot is redundant, the location of the singularities is found using the following formula: $det(\boldsymbol{J}\boldsymbol{J}^T) = 0$ [4,5].

If the definitions of singularities seem now to cover a large spectrum of robot topology, it is still difficult to identify them especially for parallel robots [6,7,8,9,10]. In some simple cases, analytical solutions can be found, but most of the time, numerical methods must be developed. For instance, in Refs. [11,12] a numerical computation of manipulator singularities is presented.

It seems also that singularities might be linked with the concept of aspects. This concept was first introduced by Borrel in 1986 [13] to cope with the existence of multiple inverse kinematic solutions in serial manipulators. Another attempts also have

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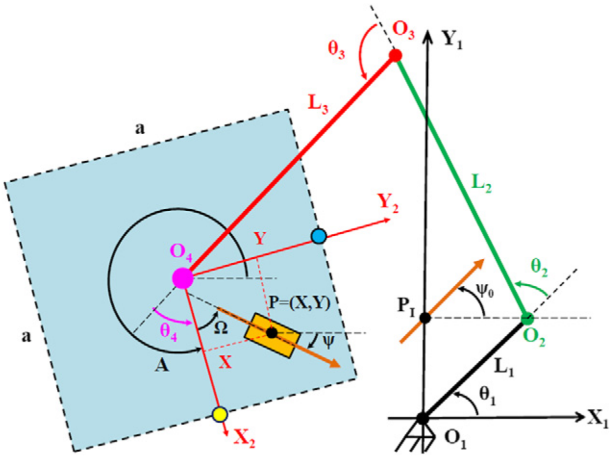

Fig. 1. Diagram of the XY-Theta platform kinematics.

been made to verify whether a path is entirely feasible in one aspect of the workspace. A t-traversable trajectory is any trajectory that could be realized in one aspect without interruption [14,3,15]. These works lead to the discovery of cuspidal manipulators that can change posture without passing through a singularity [2,16]. Malek studied the singularities in the presence of joint limits, for non-redundant robots [17,18,19] and redundant robots [20].

When the end-effector (EE) operates close to a singulary, the actuators may suffer from high velocities and torques; the control of the robot can lead to a loss of dof in the velocity space. Therefore, singularities are then more known for their drawbacks. Nevertheless they have also interesting properties, especially in the field of precision [21,22]. In Ref. [23], it is shown that the spatial resolution and repeatability of a 2R planar serial robot are much better when the EE is in the center of the workspace, i.e. close to a singularity. In this case, the lever arm between the EE and the joint center is short so the angular uncertainty is not amplified. This property is used to design new robots' kinematic architectures and control for enhanced precision performances [24]. Nevertheless, it is necessary to identify properly the loci of singularities to avoid them during the planning and control stage.

In Section 2, an innovative XY-Theta platform is presented in details with two working modes: a coarse positioning mode using four motorized joints and a fine positioning mode using only the 1st and 2nd axes. The platform design takes into account the benefits of singularities for enhanced precision performances. In Section 3, the Jacobian matrix of this platform is calculated and its singularity configurations are analyzed. In the fine positioning mode, the 1st axis should be close to $\pi/4$, so in a first approach, the 1st axis is considered constant and the robot non-redundant. The singularity loci are then drawn. If the desired trajectory cannot avoid the singularity, it is then possible to release the constraint on the first axis and use the redundancy to avoid the singularity. In this case, the robot is redundant and the singularity analysis is also performed. In Section 4, the robot manipulability index is defined and computed. It will be used to check the results of Section 3 and also characterize the distance from singularities. Finally, the conclusions are presented in Section 5.

2. XY-Theta platform kinematics and control modes

2.1. Overview of the platform structure

The XY-Theta platform consists of a 300[mm] × 300[mm] square and is held by a redundant kinematic chain of four motorized vertical revolute joints, with respective angles $\theta_1, \theta_2, \theta_3$ and θ_4 displayed in Fig. 1. The first arm length is $L_1 = 30$[mm]. The second and third arm lengths are $L_2 = L_3 = 120$[mm]. The prototype is displayed in Fig. 2.

This robot is designed to achieve precise positioning in the vicinity of a specific point of the workspace named P_I“ point of interest”. $O_1P_IO_2$ is an isosceles, right-angled triangle when $\theta_1 = \frac{\pi}{4}$. The choice of this value is done so that the lever arm lengths between the first and second axes O_1 and O_2, and P_I are minimum. These lengths can be calculated as: $O_1P_I = O_2P_I = L_1\cos(\frac{\pi}{4}) = \frac{30}{\sqrt{2}} = 21.21$[mm] and they are 6 times shorter than the second or third arm lengths of 120[mm]. Moreover, the micro-movements induced by a micro-rotation of θ_1 and θ_2 are orthogonal.

There is an additional motorized vertical prismatic axis, which can hold a tool, for instance a gripper to grasp a workpiece. The vertical axis projection on the base lies at P_I. The robot is designed so that any workpiece on the platform can be picked and placed somewhere else on the platform with the desired orientation. Thus the operational space enables three degrees of freedom for the workpiece. To characterize the dof of the workpiece, the operational coordinates $\boldsymbol{x} = (X, Y, \Omega)$ of the workpiece are given in the platform frame $(O_4X_2Y_2)$. $(O_1X_1Y_1)$ is the world frame coordinate system.