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A new design methodology for four-bar linkage mechanisms based on derivations of coupler curve

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ABSTRACT

This paper presents a new design methodology for crank-rocker four-bar linkages based on differential objective functions. Firstly, to avoid obtaining a mechanism with an unintended trajectory shape, we classified the trajectory of the linkage into four types of shapes. Two-step optimization for a specific shape type is proposed. In the first step, the shape of the trajectory is obtained by minimizing the root mean square error (RMSE) between the slope (first-order derivative) of the coupler point and that of the target trajectory. The size of the trajectory is then determined by minimizing the RMSE between the change in angle of slope (second-order derivative) of the coupler point and that of the desired trajectory. This new approach has three advantages: i) the desired trajectory can be set as a continuous and closed loop, ii) the optimal solution can be obtained without the possibility of generating a mechanism with an unintended coupler curve, and iii) the method can take account of the velocity of the output for each section with a constant input velocity. Three case studies were conducted to verify the advantages of the new approach based on a new index called the goodness of traceability.

1. Introduction

Four-bar linkages are widely used in mechanical devices due to their simple structure, ease of manufacturing, and low cost. The path synthesis of planar four-bar linkages has been studied with a variety of methods during the past 60 years [1–7]. The methods in these studies can be classified as graphical, analytical, or numerical.

Hrones and Nelson used a graphical method that involves an atlas of coupler curves [1]. They developed a four-bar linkage atlas with almost 7000 coupler curves. Similarly, Zhang et al. [8] proposed an atlas of five-bar geared linkage coupler curves. Alternatively, other graphical methods involve drawing by hand [9]. Graphical methods are quick and straightforward, but the accuracy of these approaches is limited due to drawing error, which can be critical for the design of precision mechanical devices. Also, because of the complexity of obtaining solutions with a reasonable result, the geometric construction may have to be repeated many times.

Analytical approaches were first addressed by Sandor [2] and by other researchers thereafter [10,11]. Methods for finding suitable four-bar linkages that can precisely trace desired precision points analytically have been developed, and the number of desired precision points has increased from four to nine. Once a mechanism is modeled mathematically and coded for a computer simulation, parameters such as the lengths of each link are easily handled to create new solutions without further programming. However, there is no analytical solution to the general problem of four-bar linkage synthesis for more than nine target points.

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Thus, this methodology cannot be applied for the design of four-bar linkages whose coupler point can trace a large number of target points or continuous and closed loops. This problem may be solved using a numerical method.

There are two types of numerical method. One is using numerical atlas database of coupler curve with Fourier series method [12,13,14]. The other method is optimizing parameters to minimize an objective function, and obtain the solution numerically. The most widely used objective function is the tracking error (TE), which is defined as the sum of the square of the Euclidean distance between the desired points and the obtained coupler points. To the best of our knowledge, the first to address this objective function was Han [3]. Due to the ease of calculation, TE has been used in various studies [5,15,16]. However, using TE for the objective function could generate unintended trajectory shapes. Furthermore, when the input angular velocity is constant, it is not easy to take account of the velocity of the coupler point that traces the trajectory of the four-bar linkage.

To overcome these disadvantages, this paper introduces a new numerical approach for the design of a four-bar linkage. Classifications are also presented for the possible trajectory shapes of a crank-rocker four-bar linkage: elliptical, semi-elliptical, crescent, and intersectional. These classifications were used to figure out the first and second-order derivatives (slope and change in angle of slope) of the coupler point, which reflect the characteristics of each shape type and the size of the entire trajectory. The root-mean-square error (RMSE) of these derivative values between the desired and obtained trajectories is used as the objective function.

This method has three advantages compared to conventional numerical approaches. First, using the derivative of the trajectory, which is a continuous function, the desired trajectory can be set as a continuous and closed loop. In contrast, methods that minimize TE require points for the desired trajectory. Second, if the four-bar linkage is designed to follow the derivative value profile according to the input angle, the method can obtain the optimal solution without the possibility of generating an unintended shape. Finally, by adjusting the interval of two peak points of the derivative profile of the desired trajectory, the method can take account of the velocity of each section of the coupler curve with constant input velocity.

The performance of the numerical method was investigated using a new index called the goodness of traceability (GT). GT is defined as TE as a function of the input angle of the four-bar linkage. This allows the shape and velocity to be considered simultaneously. GT can be used to compare the performance of each method objectively.

This paper is organized as follows. The trajectory classification of the four-bar linkage is presented in Section 2. Section 3 presents the new design method, and Section 4 presents the performance of the method in comparison to a conventional method. The GT index is also demonstrated in this section. Discussions and a conclusion are given in Section 5 and Section 6, respectively.

2. Classification of trajectories of crank-rocker four-bar linkage coupler point

The trajectories of the coupler point of a crank-rocker four-bar linkage need to be classified to gain insight about them. The shape classifications are explained, and the mathematical properties of each shape type are described.

2.1. Coupler point of crank-rocker four-bar linkage

To form a crank-rocker four-bar linkage, the relationships between each link length need to satisfy the Grashof conditions [17]:

$$T_1 = l_4 + l_2 - l_1 - l_3 > 0 \quad (1a)$$

$$T_2 = l_3 + l_4 - l_1 - l_2 > 0 \quad (1b)$$

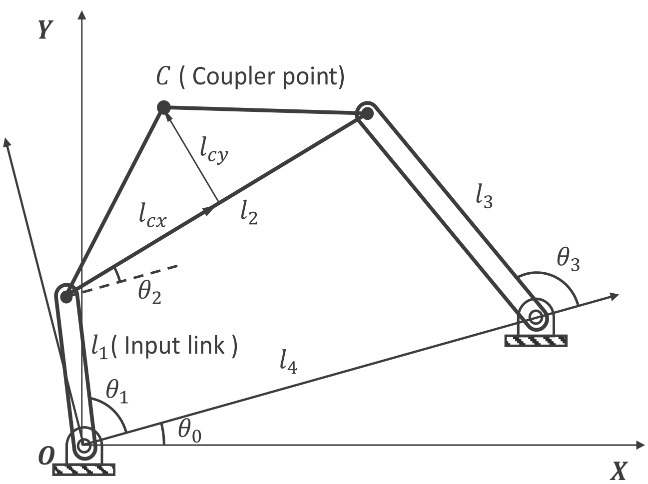

Fig. 1. Four-bar Linkage mechanism in a global coordinate system.