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Off-line programming of six-axis robots for optimum five-dimensional tasks

J. Léger *, J. Angeles

McGill University, Montreal, Canada

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ABSTRACT

Five-degree-of-freedom (five-dof) tasks are of particular interest in industry, since machining, arc-welding and deburring operations all fall into this category. These tasks, normally conducted with industrial six-dof robots, render the robot functionally redundant. Upon exploiting this redundancy to minimize the condition number of the Jacobian matrix, it is expected that the accuracy of the performed task will be increased. Traditional methods for redundancy-resolution are normally used to solve the more frequent intrinsic redundancy; however, they are not applicable to functional redundancy. Five-dof tasks are formulated using an approach that leads to a system of six velocity-level kinematics relations in six unknowns, with a 6×6 Jacobian matrix, of nullity 1, which reflects the functional redundancy of the problem at hand. To resolve the foregoing redundancy, a method based on sequential quadratic programming (SQP) is proposed. A novel method to compute the gradient of the condition number is also discussed, as it is a key element for finding the posture of minimum condition number using a gradient method. An example then shows how the SQP algorithm can be applied to offline robot trajectory-planning for five-dof tasks. In this example, a comparison is also made between the quasi-Newton and the Newton–Raphson methods to find the posture of minimum condition number for the robot. This is an essential step in finding the trajectory of minimum condition number.

1. Introduction

The growing popularity of robots for manufacturing operations has brought about the need of robots with higher accuracy than what is currently available. The need for accurate robots, however, has been reported not only for manufacturing tasks. Recently, a robot for surgical tasks was proposed [1]. For high-accuracy robots, not only the components of the robot must be precisely built and assembled, but the path-planing and control algorithms must also be devised so as to improve accuracy. In some instances, the robot might be redundant, in which case more *degrees of freedom* (dof) are available than needed; therefore, a secondary task can be accomplished. This is the case in arc-welding and machining operations involving an axisymmetric tool [2,3,4,5].

In parallel manipulators, redundancy can be used to avoid singularities in the workspace of the robot [6,7]. Upon avoiding singularities, the dexterous workspace is increased [6]. This work focuses on serial robots; however, the ideas presented here could be adapted to parallel manipulators.

Two types of redundancy have been identified, *functional* and *intrinsic*. To properly define these two, three spaces come into play: the *joint space* $\mathcal{J}$ is the space of joint variables; the *operational space* $\mathcal{O}$ is the reachable Cartesian space of the end-effector;

* Corresponding author.
E-mail addresses: jeremie.leger.1@etsmtl.net (J. Léger), angeles@cim.mcgill.ca (J. Angeles).

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and the *task space* $\mathcal{T}$ is the Cartesian space of the task. For the robot to be able to accomplish a task, the relations below should be observed:

$$\mathcal{T} \subseteq \mathcal{O} \tag{1}$$

$$\dim(\mathcal{T}) \leq \dim(\mathcal{O}) \leq \dim(\mathcal{J}). \tag{2}$$

Serial robots that have a joint-space dimension greater than their operational-space dimension are termed intrinsically redundant, their degree of redundancy, r_i, being defined as

$$r_i = \dim(\mathcal{J}) - \dim(\mathcal{O}). \tag{3}$$

On the other hand, *functionally redundant robots* have an operational-space dimension greater than their task-space dimension; this degree of redundancy, r_f, is defined as

$$r_f = \dim(\mathcal{O}) - \dim(\mathcal{T}) \tag{4}$$

the total degree of redundancy thus being

$$r_t = r_i + r_f. \tag{5}$$

Intrinsic redundancy has been discussed extensively in the literature [8,9]. Redundancy-resolution algorithms are generally based on the generalized inverse of the rectangular Jacobian matrix using the gradient-projection method (GPM), as first reported by Liégeois [10]. Crucial to the GPM, the Jacobian matrix must be of $m \times n$, with $m > n$, to be able to exploit the Jacobian matrix null space. This is, however, not the case for all types of redundancy, as pointed out by Sciavicco and Siciliano [11].

Functional redundancy, on the other hand, can yield a nonsingular square Jacobian matrix. This is the case of machining and welding robots. To solve the functional redundancy, Baron proposed to insert a virtual joint [12], thus adding a column to the Jacobian. Using the modified Jacobian, the GPM can then be used to resolve the redundancy. A more geometrically intuitive method, termed the *twist decomposition algorithm* (TWA) [3], makes use of projection matrices in the operational space to find the null space of the Jacobian. TWA was shown to be faster than other methods for functional redundancy-resolution [13]. Here a redundancy-resolution scheme using sequential quadratic programming (SQP) [14] via the orthogonal decomposition algorithm (ODA) [15] is proposed.

SQP has been shown to work as a redundancy-resolution algorithm, but only for intrinsically redundant robots [16]. The novelty of the work reported here lies in applying SQP to a functionally redundant robot for which the null space of its Jacobian is empty.

As a secondary objective, performance criteria have been developed to: avoid obstacles [17]; avoid joint limits [12,18]; minimize joint velocities and joint torques [19]; increase power transmission [4]; avoid singularities [12,20]; or a combination of multiple criteria [21,2,12]. In this work, the focus is on avoiding singularities.

In singularity avoidance, the two most popular performance criteria are manipulability [22] and condition number [20]. Manipulability suffers from not being able to measure distance from singularity, but rather only being able to identify when the robot is at a singular posture, i.e., when its manipulability is null. The condition number, on the other hand, being a measure of distance from singularity [23], it will be used as a performance index. By increasing distance from singularity, the propagation of joint error to the end-effector pose error is reduced, thus increasing accuracy. The condition number, moreover, depends on the matrix norm chosen to define it; however, its significance is independent of the chosen norm, since the different norms are monotonically related [23]. Given that SQP requires the objective function to be twice continuous differentiable, the Frobenius norm is preferred, as it is an analytical function of its argument. This condition number guarantees the continuity of the first and second derivatives.

To calculate the condition number, a normalized Jacobian matrix must be used in order to render the robot Jacobian dimensionally homogeneous. The characteristic length, introduced by Angeles [24] and defined as the length that minimizes the condition number of the dimensionally homogeneous Jacobian, is one way of normalizing the Jacobian matrix i.e., of rendering the robot Jacobian dimensionally homogeneous. The characteristic length being based on the condition number of the normalized Jacobian matrix, these two concepts go hand in hand, as one is defined based on the other. Gosselin [25] proposed a different approach, by redefining the Jacobian matrix using only point velocities. In this approach, multiple points of the end-effector are used to fully define its motion, instead of the natural definition based on the velocity of one point of the end-effector and the angular velocity of the same. The idea of using multiple points was then further investigated for parallel manipulators [26,27]. In this approach, caution must be exerted in the selection of the points, as their location influences the condition number.

In this paper, an extension and in-depth revision of a conference paper [28], the authors investigate — see Section 2 —the unconstrained condition-number minimization through applicable algorithms, while providing a discussion on the pertinent normality conditions. In Section 3, the SQP method via the ODA for use in the functional redundancy-resolution is developed for