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A new curve for application to the rotor profile of rotary lobe pumps

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This paper proposes a new type of curve, an elliptical roulette, which it initially applies to rotary lobe pumps. Once the new rotor profile has been mathematically modeled using the principle of gearing, equations can be derived to assess the presence or absence of undercutting in the tooth profile. The effect of the new profile properties on pumping performance is evaluated using a specially developed three-dimensional fluid analysis model. The proposed curve is tested using six novel rotor profiles based on the elliptical axial ratio parameter λ (a shorter axis divided by a larger axis). These six new cases and a traditional case are analyzed under the same volume and clearance conditions and their differences compared. The results show that a smaller elliptical axial ratio design produces better flow characteristics. In particular, an elliptical axial ratio smaller than 0.6 not only achieves high discharge efficiency but allows vibration and noise to be controlled by the flow rate fluctuation coefficient, which approximates the traditional indicator. The proposed curve can thus serve as a useful reference for pump or rotary fluid machine design.

1. Introduction

Rotary lobe pumps, such as blower pumps, vacuum pumps, and hydraulic pumps, are fluid-extracting apparatuses used widely in industry. A type of positive displacement pump, they use two or more rotating lobes, in a specially shaped cylinder, that intermesh with each other using timing gears to draw fluid in from one side to deliver to the other. These two rotors rotate in opposite directions with a constant gear ratio and maintain only tiny clearances during operation.

The majority of studies relevant to this paper focus on the geometric design and rotor analysis of the Roots type rotary pump. For example, Litvin and Feng [1] developed a geometry for screw Roots blowers in which the addendum tooth profile is a single circular arc. They then studied the conjugation of surfaces or rotors, the synthesis of rotor surfaces with two contact lines, and the avoidance of singularities. Shung and Pennock [2] proposed a unified compact equation describing the geometry and geometric properties of the different types of trochoid and the geometric properties of a conjugate envelope. Demenego et al. [3] developed a tooth contact analysis (TCA) computer program and discussed avoidance of tooth interference and rapid wearing through modification of the rotor profile geometry of a cycloidal pump whose one pair of teeth is in mesh at every instant. Wang et al. [4] proposed certain design constraints for five-arc Roots vacuum pumps, including the conditions for avoiding tooth undercutting and carryover. Mimmi and Pennacchi [5–8] examined rack generated rotors with three lobes and considered profiles comprising an arc, an involute, and an epitrochoidal arc with a constant trochoid ratio. Using these profiles, they analyzed admissible compositions that would avoid unreasonable rotor profiles and undercutting. They also investigated the load dynamics on rotors of a three-lobe Roots blower and found that increasing the reservoir and using helical shaped rotors improves dynamic load regularity and decreases system vibrations.

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Nomenclature

a	elliptical larger axis
b	elliptical shorter axis
r_p	radius of the base circle (radius of the pitch circle)
λ	elliptical axial ratio parameter (a shorter axis divided by a larger axis)
n	number of lobes
S_i	coordinate system i where $i = 1,2,f$
$\mathbf{R}_i$	position vector of rotor i represented in coordinate system S_i, $i = 1,2$
$\mathbf{M}_{ij}$	coordinate transformation matrix form system j to system i
O_i	origin of the coordinate system S_i, $i = 1,2,f$
ϕ	rotation angle
ψ	parameter of the elliptical profile
θ	parameter of the base circle (or pitch circle)
ζ	normal angle at a point on the rolling ellipse
$\mathbf{N}_1$	normal vector represented in coordinate system S_1
$\mathbf{V}_1^{(12)}$	sliding velocity represented in coordinate system S_1
η	area efficiency
QI	increment of average flow rate
I	coefficient of flow rate fluctuation

Liu et al. [9] and Tong and Yang [10–12] proposed a novel design method, the derivation function method, used mainly to design rotor profiles for high-sealing lobe pumps and synthesize lobe pump profiles with given flow rate functions. In particular, this method can be used to reshape original pitch pairs to obtain the desired profiles of the generated pairs. Subsequently, Hwang and Hsieh [13,14] proposed a new rotor profile with a variable trochoid ratio and investigated how to achieve high volumetric efficiency and high sealing. Valdès et al. [15] used a semi-empirical Knudsen–Dong law to calculate internal leaks in dry Roots vacuum pumps. Their method predicts the conductance of each clearance within a few percent in a wide pressure range covering the transient flow regime. Burmistrov et al. [16] developed an angular coefficients method for calculating channel conductance in Roots pumps based on the similarity between laws of radiation and diffusive reflection of molecules. This method enables accurate calculation of the conductance of Roots pump channels for different rotor positions.

For flow simulation, Voorde et al. [17] applied the fictitious domain method to study a three-blade lobe pump and tooth compressor. Houzeaux et al. [18] presented a finite element method for the simulation of rotary positive displacement pumps. This method, however, although it solves leakage problems in the gap and gear intersection, has two drawbacks: First, generating configurations is tedious if the geometry is complicated. Second, one Navier–Stokes solver must solve each configuration flow, meaning that simulations using many configurations require a huge amount of memory. Strasser [19] used a commercial computational fluid dynamics (CFD) solver, FLUENT, to simulate the gear pump. Similarly, Huang and Liu [20] used the renormalization group k-ε model, PISO algorithm, and second-order upwind difference scheme to solve the governing equations of an involute-type three-lobe positive discharge blower. All these studies [18–20], however, focused on the involute external gear pump, and most used primarily a two-dimensional numerical approach by means of computational fluid dynamics or CFD tools.

What the above literatures does make clear, however, is that volume efficiency and leakage level are the key points in pump design, meaning that topics related to the method of rotor geometric design and flow analysis are important. This present paper therefore develops a new geometric design method, one based on an elliptical roulette (the path traced by a point attached to an ellipse as it rolls without slipping along a fixed circle), which it then applies to the rotor profile of rotary lobe pumps.

To assess the performance of this new curve, a three-dimensional fluid analysis model of rotary lobe pumps is constructed that enables analysis of pump flow characteristics. Here, the effects of this new design on fluid are exemplified using a two-lobed rotor. The results thus provide a useful reference for the design of rotary lobe pumps. The next sections outline and discuss the mathematical model of the new rotor profile, the undercutting analysis, and the fluid analysis models.

2. Mathematical models of rotors and undercutting

2.1. Mathematical model of the new curve

As Fig. 1(a) shows, the equation of ellipse can be expressed as

$$\mathbf{R} = [x(\psi),\ y(\psi)] = [-a\cos\psi,\ b\sin\psi]. \tag{1}$$

When the rolling ellipse rotates counter-clockwise around the circumference of the base circle in a pure rolling motion, the point p on the rolling ellipse will trace an elliptical path. As Fig. 1(b) illustrates, because the rolling motion is pure, the length of curve 12 on the