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A robust model for determining the mesh stiffness of cylindrical gears

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ABSTRACT

A model for determining mesh stiffness of cylindrical gears is proposed using a combination of finite element method (FEM) and local contact analysis of elastic bodies. The deformation at each contact point is separated into a linear global term and a nonlinear local contact term. The global compliances are obtained using an even mesh technique and substructure method of a three-dimensional finite element analysis to make the deformation of a global term insensitive to the twist of gear structure under different gear basic parameters, while the local contact deformations are derived through an analytical line-contact formula deduced from the Hertzian contact theory. The time-varying mesh stiffness and load distribution can be well predicted in this model. It is proved that the mesh stiffness calculated from the proposed method is in close agreement with that from published formulae, but with less time consumption and improved steadiness compared with conventional FE models using contact elements. The sensitivities of total mesh force, gear basic parameters and body parameters on mesh stiffness are investigated and discussed.

1. Introduction

It is well known that the fluctuation of time varying mesh stiffness is one of the main excitations that causes unwanted vibration and noise of gear sets. The influence of mesh stiffness on vibration has been analyzed in previous studies for different types of gear system [1–4]. Determination of mesh stiffness and transmission error has always been the prior issue in gear dynamic analysis. Therefore, to obtain a more thorough understanding of the influences of different gear parameters on mesh stiffness of gears is essential in the design improvement of gear system.

The deformations of two engaged gears are crucial in the calculation of mesh stiffness. The total deformation of gears contains the global bending and shearing deformation of gear tooth and gear body, and the local contact deformation near the contact zones. As the bending and shearing deformation is linear with the applied load, while the contact deformation is nonlinear with the applied load, it will lead to nonlinearity of total deformation with the applied load, and so will the mesh stiffness.

The early studies for mesh stiffness mainly focused on the spur gears. Weber [5] and Cornell [6] calculated the deformation of spur gears using a variable cross-section cantilever beam to simulate the gear tooth. The bending and shearing deformation of the cantilever as well as the contact deformation were calculated using analytical formulae of material mechanics and contact mechanics. In the 1980s, Terauchi and Nagamura [7,8] proposed a method of conformal mapping of complex variables in plane elasticity to calculate the deflections of various spur gear teeth.

Because the transmitted loads are not evenly distributed along the lines of contact for helical gears, the calculation of tooth deflection for helical gears has to be treated as a three dimensional problem. A widely used model to calculate the load distribution

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Nomenclature

α_{ai}	Transverse pressure angle at tip circle of gear i
α_n	Pressure angle
α_{tp}	Transverse operating pressure angle
β	Helix angle at reference circle
β_{bi}	Base helix angle of gear i
δ	Static transmission error
Δ_i	Relative elastic deformation of the contact point pair i
ΔK	Fluctuation value of K
ΔL	Fluctuation value of total contact length
ε_i	The initial separation distance between contact points i
$\{\varepsilon\}$	Vector of initial separation distances
ε_α	Transverse contact ratio
ε_β	Overlap ratio
ε_γ	Total contact ratio
λ_{Gij}	Global deformation of gear pair along the normal direction of point i when a unitary normal load is applied at point j
λ_{Li}	Equivalent local contact compliance of point i
$[\lambda]$	Equivalent total compliance matrix
$[\lambda_G]$	Global compliance matrix of the gear pair
$[\lambda_L]$	Equivalent local contact compliance matrix of the gear pair
$[\eta_G^i]$	Global compliance matrix of contact points for gear i
$[\eta_G^{FE}],[\eta_L^{FE}],[\eta_T^{FE}]$	Global, local and total compliance matrices of finite element nodes on the interested tooth surface
ν_i	Poisson's ratio of gear i
B	Face width
b_s	Web thickness
C_n	Bottom clearance coefficient
C_R	Gear blank factor in ISO6336
eps	Given convergence tolerance in iteration
d_0	Central hole diameter
d_{in}, d_{ex}	Internal and external diameter of gear web
d_f	Diameter of root circle
E^*	Equivalent Young's modulus
E_i	Young's modulus of gear i
F_j	Contact force at contact point j
$\{F\}$	Vector of unknown contact forces
FE	Finite element
FEM	Finite element method
h_{an}	Addendum coefficient
k_i	Stiffness of contact point pair i
K	Time varying mesh stiffness in per unit face width [N/(mm · μm)]
K'	Time varying single stiffness
K_m	Mean value of K
K_w	Time varying mesh stiffness of the gear pair [N/m]
l_i	Length of each subsection contact line
L	Time varying total length of contact lines
L_m	Mean value of total contact length
m_n	Normal module
n	Number of potential contact points
N	Integer part of ε_γ
n_a	Number of active contact points
n_c	Total number of contact points on the interested tooth surface
n_{FE}	Total number of finite element nodes on the interested tooth surface
n_k	Number of contact points on contact line k
$n_{profile}$	The section number of finite element meshes along tooth involute
n_{width}	The section number of finite element meshes along face width
p_{bt}	Transverse base pitch
$o_M - x_M y_M$	Coordinate system on plane of action
$o_i - x_i y_i z_i$	Coordinate systems for contact points on tooth surface of gear i