Mechanism and Machine Theory 87 (2015) 115–130

Contents lists available at ScienceDirect

Mechanism and Machine Theory

journal homepage: www.elsevier.com/locate/mechmt

Efficiency analysis of two degrees of freedom epicyclic gear transmission and experimental validation

Chao Chen *, Jiabin Chen *

Department of Mechanical and Aerospace Engineering, Building 31, Monash University, Clayton, Victoria 3800, Australia

ARTICLE INFO

Article history:
Received 29 June 2014
Received in revised form 4 November 2014
Accepted 30 December 2014
Available online 21 January 2015

Keywords:
Efficiency analysis
Epicyclic gears
Lagrange multiplier
Virtual power
Complete analysis

ABSTRACT

A complete method based on constraint analysis and virtual power is developed in this work. The method is applied to a two-dof epicyclic gear transmission for efficiency modelling and prediction. The power flow diagrams are constructed to obtain possible power flow patterns and to verify the balance of normal power flow and virtual power flow within the system. The theoretical efficiency formulas were derived for the full working range, and further verified by experimental data. It was found that two local loss factors are always coupled together in this device under test, and the same speed ratio may yield different efficiencies depending on the directions of the input angular velocities.

1. Introduction

Epicyclic gear transmissions are widely utilised in various industrial applications including robotic arms [1], hybrid vehicle power transmissions [2] and turbine generators [3]. Analysing and enhancing the operational efficiency of gear transmissions are important to design optimisation and control. Recent study [4] showed that the input-split HEV (Hybrid Electric Vehicle) should be controlled by considering the transmission efficiency as well as the engine efficiency to optimise the overall system efficiency. Research in Ref. [5] emphasised that power flow and efficiency analysis enable transmissions to achieve high efficiency and low energy consumption by avoiding power circulation.

Despite the advantages of epicyclic gear trains such as compact structure, lightweight and high power density, they may have relatively low efficiency compare to simple gear systems [6]. The principle power losses in gear trains are caused by sliding friction between meshing gear tooth surfaces, churning of lubrication oil and friction in shaft support bearings [7]. Ref. [8] indicates that in low speed gearing system of an aircraft, the gear mesh losses have dominant impacts on the overall performance of the gear system. Here, only power loss due to gear mesh is considered. Our previous work [9] indicated that immense latent power losses in gear meshes could significantly reduce the efficiency of an epicyclic gear train.

Many results have reported by studying simple and compound epicyclic gear systems. A general algorithm was reported in Ref. [6] to determine efficiency of spur-gear trains. The efficiency prediction model in Ref. [10] incorporates load dependent power losses and inertia effects, and can be readily applied in changing speed and load conditions. However, more accurate bearing loss model, meshing loss model and churning loss model are required to achieve better overall prediction. Kahraman et al. [11] proposed a general formulation for kinematic analysis and power flow analysis. Salgado and Del Castillo [12] developed power flow maps to analysed efficiency of EGTs. Ref. [13] introduced a prediction model, which accounts for load distribution and friction factors. The average mechanical

* Corresponding authors. Tel.: +61 3 99053403.
E-mail addresses: chao.chen@monash.edu (C. Chen), jiabin.chen@monash.edu (J. Chen).

http://dx.doi.org/10.1016/j.mechmachtheory.2014.12.017
0094-114X/

efficiency was then obtained by averaging the sum of the instantaneous efficiency of an entire mesh circle. Their experiments demonstrated that the measured values are within 0.1% deviation compared to model predictions. Furthermore, a method based on virtual power and virtual power ratio was introduced in Ref. [14]. Pennestrì, Mariti and Valentini [15] used a technique based on the graphic representation of kinematic chains to illustrate all possible configurations of hybrid vehicle transmissions and their associate efficiencies.

Ref. [16] demonstrates a technique to formulate the efficiency equations of a 2K-H differential gear train, which can be applied easily. This method adopts torque balance and gear teeth ratio concept to estimate the efficiency for different types of configurations. Also, three 2K-H epicyclic gear trains were built with the graph theory to investigate the power losses and the transmission efficiency [17]. The work was based on the principle of energy conservation together with integrated energy transfer equations along the interaction model. Wang and Cui [18] proposed a simple algorithm to determine the power flow direction and estimate the power circulation based on transmission ratio of basic links in differential and transmission gear trains. However, this method does not come with an efficiency curve to assist in investigating the relationship between efficiency and gear ratio.

Yin et al. [19] established a planetary gear transmission system with a system modelling method to investigate the power loss and efficiency. The power losses and efficiency are verified from the experimental data. The systematic method with power loss formulas proves that to evaluate power losses and efficiency is more accurate and reliable than the normal meshing power approach. Yet, this method is not capable of illustrating power flow patterns and detecting power circulation. It lacks of an overall efficiency plot to present efficiency with respect to the input speed. Hence, it falls short in helping engineers on improving the existing designs.

The general approach based on constraints and Lagrange multipliers was developed in Ref. [20]. It computes the internal power flows through all links. The total power losses are obtained from the genuine power through each gear mesh. However, the assumption that the power losses at gear meshes are independent on each other is not accurate. On the other hand, the graphic method upon the virtual power ratio [21] considers the correlation among the power losses at gear meshes. Its difficulty lies on the determination and the one upon virtual power ratio should yield a better estimation of the overall efficiency. Experiments on a prototype of two-dof epicyclic gear transmission were conducted to validate the theoretical results.

Our previous work [21] was on a simplified model of the complete system while this work focuses on the complete system with three gear transmissions. This work extended our previous method on constraints [20] to complete analysis by considering the effects of power losses on branch power flows. A comprehensive power flow patterns were derived for more accurate efficiency prediction. One major finding in this work is that the total efficiency is dependent on the angle, θ (which is different from the speed ratio), the coupled local efficiency, η_{12}, and the third local efficiency, η_3. It means that η_1 and η_2 are always coupled together, which simplifies our calibration. This is the first time that the method based on constraint analysis is verified experimentally.

The rest of this work is organised as follows. The framework based on Lagrange multiplier and kinematic constraints is discussed in Section 2. The prototype of a two-dof epicyclic train is described with its power flows being derived in Section 3. The complete analysis upon virtual power ratio is conducted in Section 4. The description of experiment setups and procedures is provided in Section 5. The calibration of local loss factor μ_3 is shown in Section 6. Section 7 compares the efficiency curves of theoretical results and experimental data. Finally, Section 8 concludes the outcomes of the comparison and analysis, and proposes future work.

2. Framework of complete analysis

This method based on constraints in Ref. [20] is utilised here for a complete analysis of one prototype. Based on the information of velocities and angular velocities, the magnitudes and directions of internal and external powers can be determined in closed-form. Some basic concepts are reviewed here.

In the analysis, it is assumed that the system is working at steady states. i. e. the speeds and the output torques are constant in separate experiments. Therefore, the work of inertia forces does not need to be considered here. Other power loss sources such as bearing power loss, thermal power loss and lubricant churning loss are neglected since they are not dominant.

2.1. Method based on Lagrange multiplier and constraints

An epicyclic system can be described as a set of constraints given by

$$\boldsymbol{c}(\boldsymbol{q}) = \boldsymbol{0} \tag{1}$$

where $\boldsymbol{c} = [c_1 \quad c_2 \quad \cdots \quad c_m]^T$ is the m dimension constraint vector while $\boldsymbol{q} = [q_1 \quad q_2 \quad \cdots \quad q_n]^T$ is the n dimension vector of generalised coordinates. The space spanned by $\boldsymbol{q}$ can be named the coordinate space, while the space spanned by $\boldsymbol{c}$ can be called the constraint space. Unit basis vectors in the coordinate space and the constraint space can be written as

$$\begin{aligned}\boldsymbol{I_n} &= [\boldsymbol{x}_1 \quad \boldsymbol{x}_2 \quad \cdots \quad \boldsymbol{x_n}]\\ \boldsymbol{I_m} &= [\boldsymbol{y}_1 \quad \boldsymbol{y}_2 \quad \cdots \quad \boldsymbol{y_m}]\end{aligned}$$

where $\boldsymbol{I_n}$ and $\boldsymbol{I_m}$ are the identity matrices with n and m dimensions, respectively.

The velocity and angular velocity $\dot{\boldsymbol{q}}$ of the whole system are determined by

$$\dot{\boldsymbol{q}} = \boldsymbol{G}\dot{\boldsymbol{q}}_{\boldsymbol{I}} \tag{2}$$