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# Stability and dynamic analyses of a horizontal axis washing machine with a ball balancer

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ABSTRACT

A mathematical model of a horizontal axis washing machine with a ball balancer is built. The Floquet multipliers are employed for judging the stability of the system based on which a special continuation program considering the influence of gravity is developed. Stability analysis is carried out with stable and unstable regions above 4 Hz discovered. Influences of several parameters on variations of the boundaries between the regions are discussed. Dynamics of the spinning cycle in Stable region 1 are studied. The phenomenon that the tub's axis moves along the lateral surface of a cone is discovered based on which equilibrium of the centrifugal forces is discussed and a potential method for getting a smaller vibration angle is proposed. A ball balancer is produced for experiment with its lubrication and defects due to friction discussed. Quantitative experiment is done on a commercial washing machine with instability of the balancer, the cone shape trajectory of the tub's axis and the method of obtaining a smaller vibration angle validated.


## 1. Introduction

Low vibration is one of the most important performance indexes of a horizontal axis washing machine, thus vibration suppression of its spinning cycle has drawn a great deal of attention from researchers and producers.

Up till now, many discussions have been focused on dynamic optimization of the horizontal axis washer. Türkay et al. [1] discussed four alternative optimization formulations and implemented them using grid and sequential quadratic programming optimization methods for suspension design of a horizontal axis washing machine. Then, a nonlinear time variant rigid body dynamic model of the suspension system is derived in [2] using Newton–Euler method which was then programmed for simulation and assessed experimentally. Boyraz and Gündüz [3] derived a 2D dynamic model of a horizontal axis washing machine regarding the rotation plane and proposed a new optimization scheme based on Genetic Algorithms. Nygårds and Berbyuk [4] built a computational model of a washing machine with bottom mount suspension in Adams/View, and then defined three objective functions related to kinematics and dynamics of the washing machine and created a numerical algorithm to solve the Pareto optimization problem. Walking is another problem to which attention has been paid by researchers. Employing rudimentary dynamic models of the washing machine systems, Conrad and Soedel [5] investigated the walk phenomenon of both horizontal and vertical axis washers. Papadopoulos and Papadimitriou [6] presented a simplified 3D dynamic model of a horizontal axis portable washing machine and investigated different criteria for determining the margin of walking during the spinning cycle. Some efforts have also been taken to study motions of the drum or predict vibrations of the system using neural networks. Lim et al. [7] conducted dynamic analysis of a drum-type washing machine using a simplified dynamic model considering gyroscopic effects. Kalkat [8] developed a neural network based prediction analyzer for two types of driving schemes of washing machines (direct drive and belt-pulley drive) for vibration and fault diagnosis of motor bearings.

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In recent years, there has been a growing tendency for a horizontal axis washing machine to be lightweight, portable and high-capacity which makes vibration suppression of the spinning cycle more challenging. An effective way is using a passive vibration control method [9] such as an automatic balancer. Actually, the automatic balancer especially a liquid one has been successfully employed in vertical axis washing machines for many years, and some literatures have been published to analyze its effect. Bae et al. [10] derived a mathematical model of a hydraulic balancer in steady state condition from a whirling model of a vertical axis washing machine, and then implemented it for dynamic analysis of the spinning cycle. Chen and Zhang [11] carried out stability analysis of a vertical axis washer and found two unstable regions of the spin drying process. As for the horizontal axis washer, although much applied work has been done, even some products with automatic balancers have been manufactured, not many studies of their dynamics have been reported in literatures yet. In fact, applying an automatic balancer on a horizontal axis washer is more complex than on a vertical one, the first problem encountered is the influence of gravity, the ball or liquid in the balancer is always at the same side of the clumped clothes at the beginning which makes the starting of the spinning cycle more difficult. What's more, even if at the steady state, due to the influence of gravity, the ball may still oscillate around its equilibrium position.

This paper carries out stability and dynamic analyses of the spinning cycle of a horizontal axis washer with a ball balancer. At first, a mathematical model is built in Section 2. Then stability and dynamic analyses of the spinning cycle are implemented in Section 3 and Section 4, respectively. At last, experiments are carried out in Section 5 with some conclusions bought up in Section 6. This paper aims at the steady state of the spinning cycle and vibration control of the transient stage will be discussed in other papers.

## 2. Mathematical model

A horizontal axis washing machine is mainly composed of a motor, a tub, a drum and a suspension system. Some products employ a planar suspension structure shown in Fig. 1(a) which comprises two springs hanging on the top and two dampers bracing at the bottom. Dynamics of this kind of washer can be described by a planar mathematical model containing two or three degrees of freedom [3,5]. However, when an automatic balancer is involved in the washer and installed at the front rim of the drum, due to instability of the balancer at a low spinning speed, serious vibration may be induced and cannot be effectively suppressed by the planar suspension system. For solving this problem, some products are employing an improved suspension structure shown in Fig. 1(b) which employs four dampers supporting at the bottom. Discussions in this paper are based on this suspension system.

For describing movements of the motor–drum–tub assembly, two reference frames are established in Fig. 1(b): a global reference frame $X_rY_rZ_r$ fixed on the ground and a local reference frame $X_bY_bZ_b$ embedded in the tub. The vector $\mathbf{x} = [x \quad y \quad z]^T$ is employed to describe the position of the origin of the local reference frame $O_b$ and $\boldsymbol{\varphi} = [\alpha \quad \beta \quad \gamma]^T$ is used to represent the pose of $X_bY_bZ_b$ where $\alpha$, $\beta$ and $\gamma$ are Cardan's angles.

### 2.1. Generalized forces

This section discusses generalized forces provided by the suspension system and the balancer.

At first, we study forces induced by the suspension system. For illustrating in a uniform way, both the spring and the damper in Fig. 1(b) are described as a special spring–damper: the springs' damping coefficient is zero with the damper's stiffness coefficient obeying the same rule. It is assumed that the joint of the $j$th spring–damper at the ground side is $U_j$ and the joint at the tub side is $D_j$. The position of $U_j$ can be described in the global reference frame $X_rY_rZ_r$ as $\mathbf{s}_{uj}$ which is a constant and the position of $D_j$ can be represented in the local reference frame $X_bY_bZ_b$ as $\mathbf{r}_{dj}$. Because $D_j$ moves along with $X_bY_bZ_b$, the position of $D_j$ can be expressed in $X_rY_rZ_r$ as

$$\mathbf{s}_{dj} = \mathbf{x} + \mathbf{A}^r\mathbf{r}_{dj} \tag{1}$$

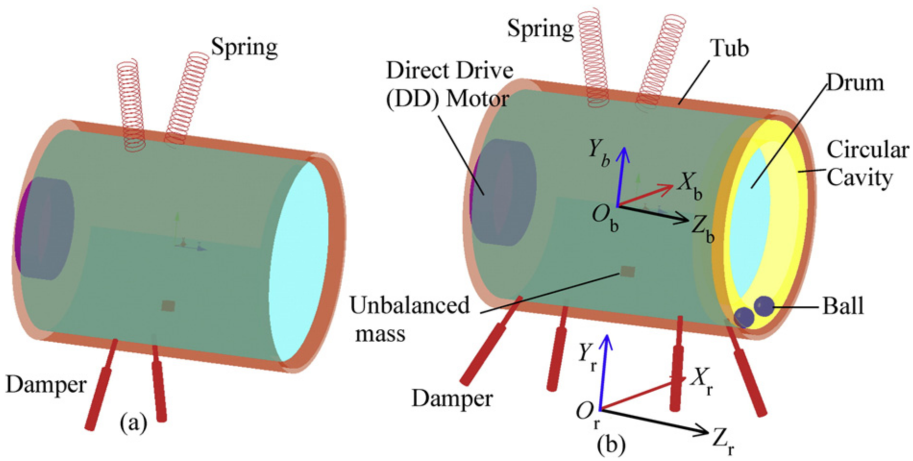


**Fig. 1.** Structure of a horizontal axis washing machine employing a traditional suspension system (a) and an improved one (b).