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Nonlinear free vibration of systems with inertia and static type cubic nonlinearities: An analytical approach

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The present paper deals with the new application of a powerful analytical method to a system with inertia and static type cubic nonlinearities. The free vibration of a mass grounded by linear and nonlinear springs in series is studied. A nonlinear ordinary differential equation with inertia and static type cubic nonlinearities represents the governing equation of the system. The new approach does not have the limitation of the traditional perturbation method which is valid for conservative systems with small perturbed parameters. An attempt has been made to provide simple approximate analytical expressions valid for small as well as large amplitudes of oscillation. The exact solution is also presented and discussed to validate the present analysis. In the paper the method for determining the equivalent rigidity for the serial connected nonlinear springs is developed.

1. Introduction

The study of vibration in complex linear and nonlinear mechanical systems is an important subject of dynamical analysis. A dynamical system such as a mass grounded by two linear springs in a series or parallel could be replaced with a spring which has their equivalents [11,19]. The system becomes complex when one of the springs behaves linearly in a series when the other one behaves nonlinearly. The nonlinear governing equation of the motion in terms of relative displacement was obtained by Telli and Kopmaz [28]. Telli and Kopmaz applied the Lindstedt method and the harmonic balance method for the governing equation of a mass grounded by linear and nonlinear springs in a series. They considered the hardening spring case in their study. Lai and Lim [17] extended both the perturbation and harmonic balance methods for nonlinear free vibration of systems with serial linear and nonlinear stiffness. They obtained higher-order approximate analytical solutions. Hoseine et al. [12] applied the homotopy analysis method to study the conservative oscillators with inertia and static type cubic nonlinearities. In recent times, substantial investigations have been done in analytical solutions for nonlinear equations without small parameters. Many of them have been used to find approximate solutions to nonlinear oscillators such as: homotopy perturbation method [20,24,26], energy balance method [1,15], variational iteration method [22,23,30], iteration perturbation method [2,18,21], Hamiltonian Approach [3,6,14,31], max–min approach [5,25,32], parameter expansion method [16,29], and other analytical and numerical methods [4,6–10,27].

In this paper Hamiltonian Approach is used to find analytical solutions for free nonlinear vibration of a system with inertia and static type cubic nonlinearities. Hamiltonian Approach was developed by He [14] and applied in different scientific works [4,13]. In this study this approach is also used. It is shown that the solutions are quickly convergent with a high accuracy and their components can be easily calculated. Some comparisons are presented to show the agreement of the Hamiltonian Approach result with the exact solution.

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2. Free vibrations of a mass grounded by linear and nonlinear springs in series connection

Fig. 1 represents a mass grounded by linear and nonlinear springs connected in a series. In the paper [28] the mathematical model of free vibrations of this system is given and widely discussed. It is concluded that due to the nonlinear properties of the springs, it is impossible to introduce one spring with equivalent rigidity which would substitute for the two springs. However, the elastic force along the springs has to be constant, i.e.,

$$F = k_1 x_1 = k_2 \Delta + \alpha \Delta^3 = k_2 \Delta + \varepsilon k_2 \Delta^3, \quad (1)$$

where k_2 and α are the coefficients associated with the linear and nonlinear portions of spring force, and ε is defined as $\varepsilon = \alpha / k_2$. From Fig. 1, Δ is net deflection of the nonlinear spring and is defined as:

$$\Delta = x_2 - x_1. \quad (2)$$

The case of $\alpha > 0$ corresponds to a hardening spring while $\alpha < 0$ indicates a softening one. Due to Eqs. (1) and (2) the relation for the elastic force distribution is obtained. While the nonlinear motion equation of the system (see Fig. 1) is

$$k_1 x_1 - k_2 (x_2 - x_1) - \varepsilon k_2 (x_2 - x_1)^3 = 0. \quad (3.a)$$

$$m x_2 + k_2 (x_2 - x_1) + \varepsilon k_2 (x_2 - x_1)^3 = 0. \quad (3.b)$$

The mathematical model is a system of a nonlinear algebraic and a nonlinear second order differential equation, as it was previously published in [28]. To solve this system of equations is not an easy task. Let us introduce new variables u and v as follows:

$$x_1 := u, \quad (4.a)$$

$$x_2 - x_1 := \nu. \quad (4.b)$$

Then, Eqs. (3.a) and (3.b) can be transformed into

$$k_1 u - k_2 \nu - \varepsilon k_2 \nu^3 = 0, \quad (5.a)$$

$$m(u + \nu) + k_2 \nu + \varepsilon k_2 \nu^3 = 0. \quad (5.b)$$

Solving Eq. (5.a) for u yields

$$u = \lambda \nu + \varepsilon \lambda \nu^3, \quad (6)$$

where

$$\lambda = k_2 / k_1. \quad (7)$$

If Eq. (6) is differentiated twice with respect to time and substituted into Eq. (5.b) one finds

$$m\left(1 + \lambda + 3\varepsilon\lambda\nu^2\right)\nu + 6m\varepsilon\lambda\nu\dot{\nu}^2 + k_2\nu + \varepsilon k_2 \nu^3 = 0. \quad (8)$$

where the dots over letters show time derivations. Eqs. (3.a) and (3.b) are reduced to solve Eq. (8).

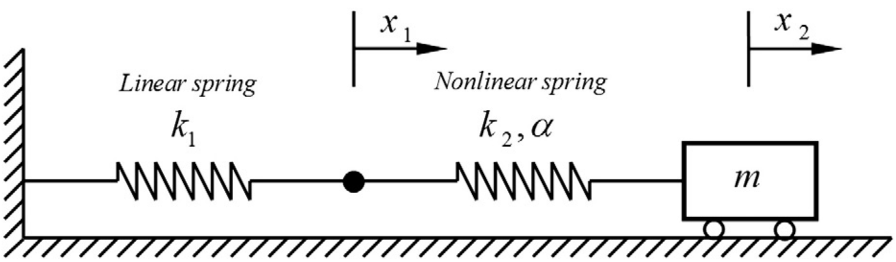

Fig. 1. Nonlinear free vibration of a system of mass with serial linear and nonlinear stiffness on a frictionless contact surface.