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Compatible reachable workspaces of symmetrical Stewart–Gough parallel manipulators

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ABSTRACT

The reachable workspace of a 6-DOF parallel manipulator developed using most existing methods is not a compatible workspace, as some of its subspaces might not be reachable through a continuous motion starting from the initial assembly configuration. This paper uses simple geometric properties and some characteristics of workspace boundaries to develop a compatible reachable workspace for a symmetrical Stewart–Gough manipulator. Equations that generate the entire workspace boundary can be easily determined by solving direct kinematics at five specific points, and the compatible workspace of a symmetrical manipulator can be developed within 10 min using a personal computer.

1. Introduction

Since the workspace of a parallel manipulator is significantly smaller than that of a serial manipulator, workspace is a very important criterion in designing parallel manipulators. An efficient method to evaluate workspace is thus very desirable in order to develop parallel manipulators with a better workspace. A reachable workspace (or maximum workspace) is a set of points that can be approached by at least one orientation of the platform. This concept is commonly used in discretization methods to develop the theoretical workspace of a Stewart–Gough parallel manipulator. If inverse kinematics yields admissible solutions (those with all the joint displacements within their joint limits) for one orientation, then the point is in the workspace. Since many boundary points of the workspace can only be approached by one orientation, it takes over 10 h to develop a more accurate boundary curve on a cross-section of the workspace [1]. This paper presents an analytical method that directly determines the boundary of the compatible reachable workspace, defined as the set of all attainable positions that can be reached through a continuous motion, starting from the initial assembly configuration.

The reachable workspace (herewith termed, workspace) developed by most existing methods (either discretization methods or analytical methods) is a theoretical workspace [1–8]. Fig. 1a shows a cross-section of the theoretical workspace and a cross-section of the compatible workspace (the region with a dark boundary) of a Stewart–Gough parallel manipulator. It shows that a relatively large subspace of the theoretical workspace cannot be reached through a continuous motion. Fig. 1b shows a configuration that can reach a point in the theoretical workspace outside the black boundary. The point, however, cannot be reached through a continuous motion starting from the initial assembly configuration, unless some limbs are disconnected somewhere on the path. The compatible workspace can be determined by performing a compatibility check on all the points inside the theoretical workspace, but this approach is extremely time-consuming, because it requires hundreds of geometrical computations to determine whether a point is

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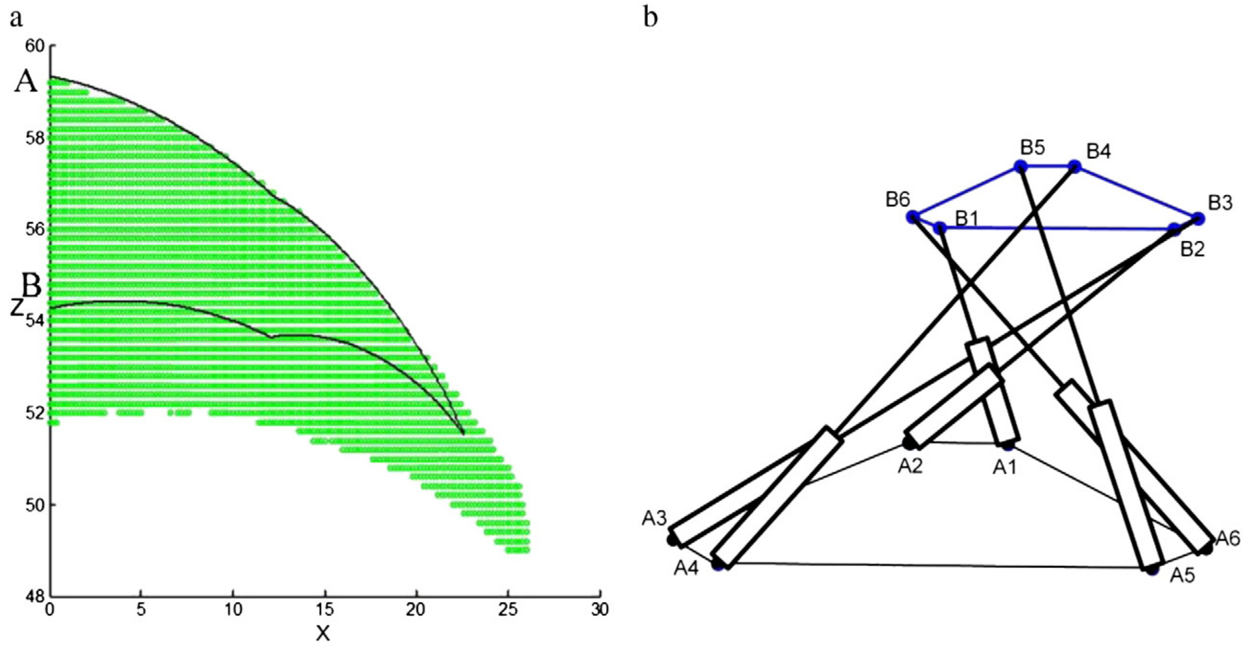

Fig. 1. Inadmissible region and configuration.

within the compatible workspace [9]. Continuously moving the tool center point (TCP) to develop the workspace boundary is a more reliable and efficient method. The boundary surface of the workspace consists of many two-dimensional patches, generated by different sets of four constraint equations. Searching for the exact equations to develop a specific patch is a very time-consuming task because there are hundreds of possible sets of equations to choose from. The boundary curve of the compatible workspace shown in Fig. 1a, in general, consists of five segments connected by extreme points A, B and three other bifurcation points on the curve. The existing method employs tangent vectors (developed from the Jacobian matrix) of all possible curves through the points to determine which set of equations generates the next segment. The equations with the maximum slope will generate the upper boundary starting from point A. For a symmetrical manipulator, the slopes of the tangent vectors of the fifteen possible curves through point A are very close because all the vectors lie on the same horizontal plane, so trial and error methods need to be used to search for the desired equations in the developing process. Therefore, it takes about 10 h to develop the boundary surface generated by actuator joint limits [10]. In this paper, algorithms that can efficiently determine the boundary curve on a cross-section of a workspace are first presented, in order to develop the boundary surfaces of many manipulators and then investigate their characteristics. Using some of these characteristics, the modified method can directly determine the equations to generate all the boundary patches using the inverse kinematic solutions at five specific points.

2. Constraint equations

The fully symmetrical Stewart–Gough parallel manipulators studied in this work are developed using the five parameters h, R_1, R_2, ϕ_1 and ϕ_2, as shown in Fig. 2. The first three parameters determine the shape and size of a manipulator and the two angles,

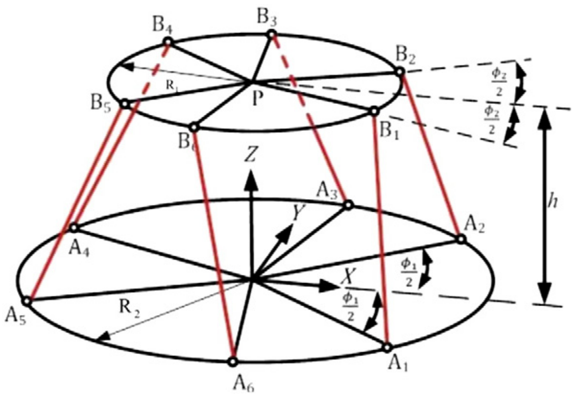

Fig. 2. Parameters for developing symmetrical manipulators.