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Optimal post-warranty maintenance policy with repair time threshold for minimal repair

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ABSTRACT

In this paper, we consider a renewable minimal repair–replacement warranty policy and propose an optimal maintenance model after the warranty is expired. Such model adopts the repair time threshold during the warranty period and follows with a certain type of system maintenance policy during the post-warranty period. As for the criteria for optimality, we utilize the expected cost rate per unit time during the life cycle of the system, which has been frequently used in many existing maintenance models. Based on the cost structure defined for each failure of the system, we formulate the expected cost rate during the life cycle of the system, assuming that a renewable minimal repair–replacement warranty policy with the repair time threshold is provided to the user during the warranty period. Once the warranty is expired, the maintenance of the system is the user's sole responsibility. The life cycle of the system is defined on the perspective of the user and the expected cost rate per unit time is derived in this context. We obtain the optimal maintenance policy during the maintenance period following the expiration of the warranty period by minimizing such a cost rate. Numerical examples using actual failure data are presented to exemplify the applicability of the methodologies proposed in this paper.

1. Introduction

From the manufacturer's perspective, the optimal maintenance policy needs to be developed to keep the system operating without failure while the warranty is in effect so that the warranty cost is minimized. On the other hand, from the user's perspective the main concern would be to find a maintenance model minimizing the maintenance cost following the expiration of warranty. In this respect, there have been a number of maintenance policies from both the manufacturer's and the user's point of view in the literature. Nakagawa [1] provides useful maintenance theory and many references on the points of view from both manufacturers and users.

Under the user's perspective, Sahin and Polatoglu [2] consider two types of replacement policies during the post-warranty period subject to certain conditions and discuss the cost incurred during the life cycle of the system. Jung and Park [3] extend Sahin and Polatoglu's [2] model by allowing the preventive maintenance (PM) activity and derive the optimal maintenance policies during the post-warranty period by optimizing the expected cost rate per unit time during the life cycle of the system. Jung et al. [4] suggest an optimal replacement policy after the replacement warranty is expired, which is based on a measure unifying both expected cost and expected downtime. Wu [5] proposes models considering human factors to conduct warranty cost analysis recently. Various maintenance policies have been studied under the warranty policy considering different conditions and situations by Yeh et al. [6], Chen and Chien [7], Chien [8] and others. Yeo and Yuan [9] study a model which incorporates the imperfect repair and extends the Yeh et al. [6] model. Later, Jung et al. [10] define the life cycle anew from the user's perspective and propose new optimal system maintenance policies during the post-warranty period, assuming the renewing warranty policy.

In their paper, the life cycle of the system is defined to start when the new system is installed and to end when the system is replaced by a new one at the expense of the user during the post-warranty period, while in many other maintenance policies, the life cycle ends when the new system installed originally fails. Other maintenance policies including those from the manufacturer's perspective are provided therein.

During the warranty period, in relation to the compensation for the failure occurrence, three basic types of warranties are usually offered: free repair–replacement warranty (FRW), pro-rata warranty (PRW) and combination warranty (CMW). See Blischke [11] and Park and Pham [12]. Under a FRW, the

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customers receive a repair/replacement warranty service for free and the PRW is such that the repair/replacement is not provided free of charge, but at a prorated cost, depending on the amount of usage or service time provided prior to its current failure. The CMW contains both features of FRW and PRW and the CMW has been considered by some papers [13–15].

In this paper, we study a renewable minimal repair–replacement (MRR) warranty model to accommodate two warranty services, repair and replacement simultaneously. Although we consider a two-factor warranty model in this study, this is quite different from the usual two-dimensional models which use the product's usage (actual time of usage) and age (calendar time) as two factors affecting the warranty policy. Repair service, which is one of the major maintenance actions, has been considered in the literature by many authors, including Park and Pham [16–18], Shafiee et al. [19]. Although the replacement models have their own usefulness in certain situations, it would be more reasonable to assume that the repair service can also be provided when the system failure occurs. Since the repair cost is usually less than the replacement cost in most situations, the manufacturer would naturally try to repair the failed product initially before providing a replacement service. The renewable MRR warranty model we consider in this study works as follows: when a system failure occurs during the warranty period, the manufacturer or the repair shop immediately starts the minimal repair for the failed system. In case, the repair cannot be completed within a pre-specified repair time limit (which is referred to as "repair time threshold"), the replacement service is provided for the user instead and the original warranty terms are renewed. Such a time limit for repair service would be set by the manufacturer for the sake of customer's satisfaction so that the repaired product or the new one can be returned back to the customer as soon as possible. In this regard, it would be worthwhile to consider a warranty policy which can provide both repair and replacement services simultaneously when the system failure occurs. Under the MRR warranty model, both repair and replacement services are simultaneously considered with the repair service time threshold pre-specified which has not been discussed so far in the literature to our best knowledge.

Once the warranty is expired, the customer maintains the system at its own expenses during a fixed length of maintenance period following the expiration of warranty and a minimal repair is conducted whenever a failure occurs. At the end of maintenance period, a replacement by a new one is taken place by the customer. Under the proposed renewable MRR warranty model, we derive the mathematical formulation to calculate the expected cost rate (ECR) during the life cycle of the system and propose a new optimal maintenance policy in terms of ECR. The uniqueness property of such an optimal maintenance policy is verified for a repairable system with an increasing failure rate.

The remainder of this paper is organized as follows. In Section 2, the research problem of the study is described. Section 3 develops the cost models under the renewable MRR warranty model, which takes both repair service and replacement service into account for each system failure. The mathematical formulas to compute the ECR's are derived as well. Section 4 discusses the methodology to optimize the ECR under our proposed renewable warranty model and the optimal solution for decision variables of the maintenance policy following the expiration of the warranty period is obtained by minimizing the ECR. Section 5 presents a numerical example and finally, concluding remarks are given in Section 6.

A. Nomenclature

pdf, cdf	probability density function, cumulative distribution function, respectively
i.i.d.	independent, identically distributed
r.v.	random variable
NHPP	Non-Homogeneous Poisson Process
ECR	Expected Cost Rate
MRR	Minimal Repair–Replacement
FMRR, PMRR	Free MRR, Pro-rata MRR
$\lambda(\cdot)$	Intensity function
T,Y	failure time and repair time, respectively
$f(t),F(t),\overline{F}(t)$	density function, distribution function, and reliability function of T, respectively
$g(y),G(y),\overline{G}(y)$	pdf, cdf and reliability function of Y, respectively
δ	fixed length of maintenance period
Ψ	warranty region which is censored by both warranty period and repair time threshold
C_r	total replacement cost during the warranty period
C_m	total minimal repair cost during the life cycle of the system
C_f	total failure cost during the life cycle of the system
c_r	unit cost of replacement
c_m	unit cost of minimal repair
c_{fm}	unit failure cost
N_R	number of replacements during the warranty period
N_ψ	number of minimal repair services within the area Ψ
N_T	total number of system failures during the warranty period
N_δ	total number of system failures during the maintenance period

B. Assumptions

(i) All the warranty claims are valid and accepted.
(ii) Length of time necessary to repair/replace the failed system is not included within the warranty period.
(iii) Failure cost, which incurs for each failure due to the stoppage of system operation, is charged to the manufacturer during the warranty period under both FMRR and PMRR.
(iv) Minimal repair is performed at no charge to the user during the warranty period under both FMRR and PMRR.

2. Problem description

In this study, the repair time and failure time are considered as two factors for the warranty analysis. There exist many systems for which the usage may not be easy to obtain. For instance, for some electric appliances such as laptop, computer and refrigerator, or for certain facilities such as nuclear reactor, their ages are readily obtainable but it may be difficult to document their exact usages. In such situations, the existing two-dimensional warranty policy may not be convenient to apply or not even applicable for those systems. On the other hand, we may easily obtain the information regarding the failure times and repair times of such systems so that these information may readily be used to adapt new warranty policy.

When the warranty expires, the system is maintained by the user for a fixed length of time, which is referred to as a maintenance period of length δ. The system is minimally repaired at each system failure during the maintenance period, and is replaced with a new one at the end of the maintenance period. Therefore, the life cycle is defined from the purchasing point of the system to the replacement time at which the system is replaced at the expense of the user after the warranty expires. The expected total maintenance cost during the life cycle is investigated under the user's point of view with consideration of the warranty period and the fixed length of maintenance period. Since all the maintenance costs incurred during the maintenance period are paid entirely by the user, the user's main