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Derivation of valid contracted graphs from simpler contracted graphs for type synthesis of closed mechanisms

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ABSTRACT

Contracted graph (CG) is a basic and effective tool for deriving topology graphs which are widely used for type synthesis of closed mechanisms. This paper focuses on the derivation of valid CGs from simpler CGs by adding edge and the identification of their isomorphism. First, the concepts of CGs are explained, and the numbers of vertices and edges in CGs are determined. Second, many different CGs are constructed from associated linkages. Based on the numbers of different edges, many CGs corresponding to the same associated linkage are grouped and the isomorphic/invalid CGs are identified and deleted. Third, many complex valid CGs are derived from simpler valid CGs or virtual CGs by adding edge. Finally, two application examples of the CGs are illustrated for type synthesis of closed mechanisms.

1. Introduction

It is has been a significant and challenging issue to create more novel mechanisms with useful functions. The topology graph (TG) is a simple and effective tool for type synthesis of closed mechanisms [1–7]. The contracted graph (CG) is a basic and effective tool for deriving various TGs [5]. In the aspect of type synthesis of mechanisms, Crossley, Dobrjanskyj, Sohn, Freudenstein, Tsai, Yan et al. proposed contracted graphs (CGs) [1–6]. Yang and Jin studied topology structure design of robot mechanisms [7,8]. Gogu [9,10] conducted structural synthesis of parallel robots using morphological and evolutionary approaches. Johnson [11] derived associated linkages (ALs) for type synthesis of planar mechanisms using determining tree and synthesized many planar mechanisms by ALs. Lu and Leinonen [12] derived the unified ALs for type synthesis of planar and spatial mechanisms and derived CGs from unified ALs using adjacency matrices. By changing types and motion orientations of joints, Yan et al. studied the configuration synthesis of mechanisms with variable topologies/topological representations and characteristics of variable kinematic joints [13,14].

Hervé proposed Lie group for classifying mechanism branches based on different joints and their orders [15]. Pucheta et al. synthesized planar linkages based on constrained sub-graph isomorphism detection and existing mechanisms [16,17]. Saxena et al. selected the best configuration for mechanisms based on kinetostatic design specifications [18]. Huang et al. studied theory of loop algebra of multi-loop kinematic chains [19]. Tuttle et al. [20,21] applied group theory to the enumeration and structural analysis of basic kinematic chains. Kong, Gosselin [22,23], Wang [24] and Tarcisio [25] et al. studied the topological synthesis and type synthesis of parallel mechanisms based on screw theory, a virtual joint, instantaneous kinematics and some wrist design requirements. Tuttle, Uicker, Shende, Rao, Yadav, Kong, Chang et al. studied the identification of isomorphic kinematic chains

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Nomenclature

μ	the complexity coefficient of mechanism
DoF	degree of freedom
F	the number of DOF of the mechanism
C	basic circle
AL	associated linkage
CG	contracted graph
TG	topology graph
DTG	topology graphs with digits
T	ternary link
Q	quaternary link
P_e	pentagonal link
H	hexagonal link
P, R, U, S	prismatic, revolute, universal, spherical pairs, respectively
n	the number of the vertices in CG
n_k (k = 3,..., 6)	the numbers of (T, Q, P_e, H), respectively
n_e	the number of the edges in CG
e_1	a single edge connected with 2 vertices
e_i (i = 2,..., 6)	the parallel edge with i single edges and 2 vertices
m_i	the numbers of e_i
n_{p2}	the numbers of B in TG for planar mechanism
n_{s2}	the numbers of B in TG for spatial mechanism

[20,26–31]. Lu et al. derived many TGs with digit and TGs for planar 3-, 4-DOF mechanisms by distributing some binary links or digits over CGs using arrays and studied relative criteria [32,33].

In fact, the type synthesis of mechanisms can be divided into the following procedures: 1) the derivations of ALs based on DOF of mechanisms; 2) the derivations of valid CGs from ALs; 3) the derivations of valid TGs from valid CGs; 4) the derivations of valid kinematics chains from valid TGs. It is well known that each of the derivation procedures is complicated and difficult. The isomorphism identification in each of the former derivations can effectively avoid a large number of unnecessary later derivations. Refs.

Table 1
Values of (n_{p2}, n_{s2}, n_3, n_4, n_5, n_6, n, n_e) in each of the 55 known ALs [12].

No.	n_{p2}	n_{s2}	n_3	n_4	n_5	n_6	n	n_e	No.	n_{p2}	n_{s2}	n_3	n_4	n_5	n_6	n	n_e
0.1	3 + F	6 + F	0	0	0	0	0	0	4.12	6 + F	21 + F	3	1	1	0	5	9
1.1	3 + F	9 + F	2	0	0	0	2	3	4.13	7 + F	22 + F	1	2	1	0	4	8
1.2	4 + F	10 + F	0	1	0	0	1	5	4.14	7 + F	22 + F	2	0	2	0	4	8
2.1	3 + F	12 + F	4	0	0	0	4	6	4.15	9 + F	24 + F	0	1	2	0	3	7
2.2	4 + F	13 + F	2	1	0	0	3	5	5.1	3 + F	21 + F	10	0	0	0	10	15
2.3	5 + F	14 + F	0	2	0	0	2	4	5.2	4 + F	22 + F	8	1	0	0	9	14
2.4	5 + F	14 + F	1	0	1	0	2	4	5.3	5 + F	23 + F	6	2	0	0	8	13
2.5	6 + F	15 + F	0	0	0	1	1	3	5.4	6 + F	24 + F	4	3	0	0	7	12
3.1	3 + F	15 + F	6	0	0	0	6	9	5.5	7 + F	25 + F	2	4	0	0	6	11
3.2	4 + F	16 + F	4	1	0	0	5	8	5.6	8 + F	26 + F	0	5	0	0	5	10
3.3	5 + F	17 + F	2	2	0	0	4	7	5.7	5 + F	23 + F	7	0	1	0	8	13
3.4	6 + F	18 + F	0	3	0	0	3	6	5.8	6 + F	24 + F	5	1	1	0	7	12
3.5	5 + F	17 + F	3	0	1	0	4	7	5.9	7 + F	25 + F	3	2	1	0	6	11
3.6	6 + F	18 + F	1	1	1	0	3	6	5.10	8 + F	26 + F	1	3	1	0	5	10
3.7	7 + F	19 + F	0	0	2	0	2	5	5.11	7 + F	25 + F	4	0	2	0	6	11
3.8	6 + F	18 + F	2	0	0	1	3	6	5.12	9 + F	27 + F	2	1	2	0	5	10
3.9	8 + F	20 + F	0	1	0	1	2	5	5.13	11 + F	29 + F	0	2	2	0	4	9
4.1	3 + F	18 + F	8	0	0	0	8	12	5.14	9 + F	27 + F	1	0	3	0	4	9
4.2	4 + F	19 + F	6	1	0	0	7	11	5.15	7 + F	24 + F	6	0	0	1	7	12
4.3	5 + F	20 + F	4	2	0	0	6	10	5.16	7 + F	25 + F	4	1	0	1	6	11
4.4	6 + F	21 + F	2	3	0	0	5	9	5.17	8 + F	26 + F	2	2	0	1	5	10
4.5	7 + F	22 + F	0	4	0	0	4	8	5.18	9 + F	27 + F	0	3	0	1	4	9
4.6	6 + F	21 + F	4	0	0	1	5	9	5.19	8 + F	26 + F	3	0	1	1	5	10
4.7	7 + F	22 + F	2	1	0	1	4	8	5.20	9 + F	27 + F	1	1	1	1	4	9
4.8	8 + F	23 + F	0	2	0	1	3	7	5.21	9 + F	27 + F	2	0	0	2	4	9
4.9	8 + F	23 + F	1	0	1	1	3	7	5.22	10 + F	28 + F	0	1	0	2	3	8
4.10	9 + F	24 + F	0	0	0	2	2	6	5.23	10 + F	28 + F	0	0	2	1	3	8
4.11	5 + F	20 + F	5	0	1	0	6	10	$n = n_3 + n4 + n5 + n6$,			$ne = (3n_3 + 4n4 + 5n_5 + 6n_6)/2$					