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Analysis of nonlinear oscillations in spur gear pairs with approximated modelling of backlash nonlinearity

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ABSTRACT

Due to international competition and strict limitations of standards regarding the noise level, investigation of gear vibration is of great importance. In this paper, nonlinear oscillations of spur gear pairs including the backlash nonlinearity is studied. Dynamic system is described through the classical single degree of freedom (SDOF) model in terms of dynamic transmission error (DTE). Using multiple scale method, forced vibration responses of the gear system including primary, super-harmonic and sub-harmonic resonances are investigated. In each case, the jump phenomenon and stability analysis are studied. In addition, the effect of dynamic and manufacturing parameters of the gear system on the DTE amplitude and consequently time responses are analyzed and interpreted physically. Results show interesting behaviors of the DTE amplitude under super/sub-harmonic resonances in comparison with the primary resonance.

1. Introduction

In the previous decades, vibration and noise of the gear systems have been studied extensively. Recently, limitations associated with the standards related to the gear system and worldwide competition increase the importance of this concern. A complete literature overview on the mathematical modeling of the gear systems have been provided by Ozguven and Houser [1]. In their research, evaluation of the dynamic factor, tooth compliance, gear dynamics, geared rotor dynamics, and torsion vibration are considered for the classification of mathematical models used in the gear dynamics. The presence of gear backlash, either caused by manufacturing errors and wear or introduced intentionally at the design stages, leads to the strong nonlinear interaction in the system dynamic equation. Therefore, various models have been proposed to consider the effect of backlash dynamics as the angular distance between the reverse tooth flanks while the forward tooth flanks remain in contact (Wang [2], Bonori and Pellicano [3]). However, the related analysis was developed based on a classical single degree of freedom model. Also, the backlash effect associated with the variable stiffness has been analyzed via the harmonic balance method by Kahraman and Singh [4].

In other modelling works, Ozguven presented a nonlinear model including the shaft and bearing dynamics [5]. Due to non-contact nature of the process, a nonlinear displacement function was considered to represent the stiffness variation (Kahraman and Singh [4,6], Blankenship and Kahraman [7]). Considering the backlash as a time varying function, Cai [8] developed a nonlinear model in which dynamic forces were set to zero during non-contact times of tooth pairs. Using the Hill infinite determinant, Amabili and Rivola [9] investigated steady-state response and stability of the spur gear pairs with time varying meshing damping through a closed form solution.

Nonlinear dynamics of the gear systems have been researched widely. In the early works, Kahraman and Singh investigated the interactions between time varying mesh stiffness and clearance nonlinearities [10]; and nonlinear dynamics of a gear rotor-bearing system with multiple clearances [11]. Parker et al. [12] accomplished a comparison between theoretical and

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experimental results of nonlinear behaviour of a spur gear pair including nonlinear mesh stiffness and backlash while Vaishya and Singh [13] considered the effect of sliding friction-induced non-linearity. Using the Floquet theory, Vaishya and Singh [14] analyzed the vibration of periodically varying gear mesh systems with coulomb friction. In the majority of these researches, fundamental issues related to the interaction of non-linear effects were studied. Models expressed by piecewise linear or non-linear equations of motion with time-periodic coefficients and external forcing were used to describe the associated dynamics. Direct integration, common numerical methodologies, the harmonic balance and shooting methods have been used for the determination of vibration response of these models.

Recently, Shen et al. [15] used incremental harmonic balance method to analyze nonlinear dynamics of a spur gear system including time varying stiffness and backlash. Similarly, Wang et al. [16] carried out the same analysis for the hypoid gear system while before that, limited investigations were done on the dynamics of the non-parallel axis gears (e.g., hypoid gear systems). Gill-Jeong [17] studied the nonlinear behaviour of spur gear pairs with one-way clutch to verify whether it is effective for reducing torsional vibration; generated by periodic engine pulsations. Thereafter, dynamic responses including softening nonlinearity and jump phenomenon were studied through numerical integration. Walha et al. [18] investigated the nonlinear dynamics of a two-stage gear system with mesh stiffness fluctuation, bearing flexibility and backlash. Unlike many previous single stage models, they developed an analytical model for the two-stage spur gears system; incorporating the deformability of the essential bodies such as the teeth, shafts and the bearings.

Through the development of nonlinear dynamics theories, nonlinear characteristics such as bifurcations, different kinds of periodic solutions, limit cycles behaviour and chaotic motions in gear dynamics have been studied. Chaotic behaviour of gear systems has been studied in some previous works. Considering the gear mesh backlash, static transmission error, and nonlinearities caused by bearing clearance and non-contact characteristics, Theodossiades and Natsiavas [19,20] analyzed the periodic and chaotic dynamics of the gear systems. In their work, the piecewise-linear technique and multi-scale method were applied, leading to the second-order approximate solutions. However, in these works [19,20] and those done by Kahraman and Singh [4,10], where the backlash was represented by truncated series expansion, it was difficult to obtain the solutions with high precisions. This problem was solved somewhat by using incremental harmonic balance method [15,16]. Litak and Friswell [21] investigated the chaotic behaviour of the gear systems with flexible shafts. Bonori and Pellicano [3] studied the chaotic behaviour of non-smooth dynamics of a spur gear pair with manufacturing errors. Coexisting solutions and bifurcations have been presented in dynamic behaviour of spur gear pairs with backlash, low damping and large finite stiffness values by Halse et al. [22]. However, in their work, a single degree of freedom model was used for a pair of meshing spur gears. Also, using analytical and finite element models, Ambarisha and Parker [23] investigated the nonlinear dynamics and chaotic behavior of planetary gears, in the presence of tooth contact loss. According to their work, nonlinear jumps, chaotic motions, and period-doubling bifurcations occur when the mesh frequency or any of its higher harmonics are near the natural frequency of the system.

Dynamic transmission error (DTE) caused by tooth deflection is another displacement-type source of vibrations in spur gear pairs (Gregory et al. [24]). In the early works, a basic theory was developed by Mark [25] to find an analytical expression for the static transmission error, which was considered the main parameter to control the gear dynamics. Thereafter, the steady state solution of the gear system for studying the parametric excitation effect on the resonance and instability conditions have been investigated (Hsu and Cheng [26], Benton and Seireg [27]). As indicated by Kahraman and Blankenship [28] and verified experimentally; many nonlinear phenomena such as sub/super harmonic resonances, tooth contact loss, bifurcation and multiple coexisting stable motions can be seen in the presence of transmission error. In addition, previous investigations such as those done by Chung et al. [29] and Smith [30] denoted the correlation between the gear noise, transmission error and vibration. In their work, the role of transmission error control and gear blank dynamic tuning for the gear noise reduction was studied. Although there is a general agreement about the nature of the phenomenon, the current understanding of gear vibration remains incomplete.

Although existing nonlinear mathematical models used to describe the dynamic behavior of a gear pair are somewhat similar to each other, they differ in terms of the excitation mechanisms and the solution technique applied. In addition, as addressed in the majority of previous works, gear backlash introduces serious difficulties in the analysis because the related equations of motion become strongly nonlinear. Consequently, complicated and irregular responses are expected to occur. Due to mentioned reasons, studying the nonlinear behavior of the gear system vibration with backlash nonlinearity in terms of transmission error is of great importance.

In this research, a new analytical methodology is applied to investigate the nonlinear oscillation of a SDOF model of spur gear pairs in terms of dynamic transmission error (DTE). Mathematical model of the problem is formulated in which the backlash nonlinearity is approximated with a third-order polynomial of DTE (cubic nonlinearity). Unlike the previous works where the nonlinearity of a gear backlash had to be modeled by a discontinuous and non-differentiable function, the proposed third-order polynomial function for backlash nonlinearity is capable of predicting more convenient and refined dynamic responses. Defining a detuning frequency parameter and using the multiple scale as one of the nonlinear perturbation methods (in MAPLE and MATLAB environments), forced vibration responses of the geared system are obtained analytically. Frequency–response equation, jump phenomenon and stability analysis are investigated in various cases of primary, super-harmonic and sub-harmonic resonances. Moreover, a comprehensive physical parametric study is accomplished to evaluate the effect of various dynamic and manufacturing parameters such as stiffness and damping values, detuning parameter and excitation amplitude on the DTE amplitude. Attractive nonlinear behavior of the DTE amplitude is observed under super/sub harmonic resonances in comparison with the primary resonance case (for various case studies while the mentioned dynamic parameters are varied).