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Posture optimization in robot-assisted machining operations

S.H.H. Zargarbashi [a,b,*,1], Waseem Khan [c,2], Jorge Angeles [d,3]

[a] Department of Mechanical Engineering, Isfahan University of Technology, Isfahan, 8415683111, Iran
[b] Department of Mechanical Engineering, McGill University, 817 Sherbrooke St. W., Montreal, Quebec, Canada H3A 2K6
[c] Jabez Technologies Inc., 5929 Rte Transcanadienne, suite 320, St. Laurent, Quebec, Canada H4T 1Z6
[d] Department of Mechanical Engineering, McGill University, 817 Sherbrooke St. W., Montreal, Quebec, Canada H3A 2K6

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ABSTRACT

This paper aims to provide a robot performance index to evaluate the effectiveness of the actuator torque and joint rates in producing a prescribed *robot gesture* during a machining operation. Most performance indices proposed in the literature are only posture-dependent; however, to properly evaluate the goodness of the force transmission when the robot performs a task, the index should be also task-dependent. The index proposed here, of the *kinetostatic* type, targets applications that call for optimizing the posture of a redundant robot. In this light, dynamic effects are not relevant to this study. The concept is formulated rigorously for the architecture most commonly found in industrial robots of the serial type, i.e., composed of six revolute joints. Furthermore, a simplified version of the same index is obtained for robots of the decoupled type, while neglecting the end-effector angular velocity and the torque applied by the cutting force. Finally, the simplified index is used to optimize the posture of a six-revolute robot while performing a five-axis machining task, thus ending up with a redundant robot for the given task.

1. Introduction

Robots are used mostly for pick-and-place operations, where only the beginning and the end of the trajectory are important, with little or no attention to the trajectory in between, as long as no collision either with objects in the environment or between the robot links themselves occur. Nowadays, the use of robots for machining operations is growing because of their flexibility to perform a broad spectrum of tasks at a lower cost when compared with machine tools. Machining operations require that the trajectory be tracked continuously. By analyzing the task and the trajectory at hand, a robot is selected to satisfy the mobility requirement with a minimum but complete joint set. If a robot with any extra joints is selected, stiffness and accuracy will be compromised. However, having extra degrees of freedom (dof) relative to the task, i.e., kinetostatic redundancy, increases the dexterity and, consequently, should improve robot performance. It is recalled that kinetostatics is the study of mechanical systems, usually in motion, under static, conservative conditions [1]. Thus, although the system is in motion, inertia effects are either excluded, or accounted for in the form of static loads, under D'Alembert's Principle. The inverse displacement analysis of machining robots has a continuous set of solutions in light of an incompletely specified end-effector (EE) pose. Indeed, the latter is specified up to the angle of rotation of the axially symmetric tool about its axis, the six-axis machining robot thus becoming redundant in the presence of the prescribed task. In this case, in addition to the EE Cartesian coordinates, a criterion can be introduced to define one set of joint-variable values uniquely.

* Corresponding author at: Department of Mechanical Engineering, Isfahan University of Technology, Isfahan, 8415683111, Iran. Tel.: +98 311 391 2625; fax: +98 311 391 2628.
E-mail addresses: amir_zargar@hotmail.com, shzargar@cim.mcgill.ca (S.H.H. Zargarbashi), waseem@robotmaster.com (W. Khan), angeles@cim.mcgill.ca (J. Angeles).*URL's:URL:* http://www.iut.ac.ir/ (S.H.H. Zargarbashi), http://www.robotmaster.com (W. Khan), http://www.cim.mcgill.ca/rmsl (J. Angeles).
[1] Tel.: +98 311 391 2625.
[2] Tel.: +1 514 333 8808#118; fax: +1 514 333 8813.
[3] Tel.: +1 514 398 6315; fax: +1 514 398 5256.

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Some well-known criteria or performance indices are (*i*) the condition number of the Jacobian matrix; (*ii*) manipulability; (*iii*) velocity ratio; and (*iv*) mechanical advantage. The condition number [2] is the upper bound of the relative roundoff-error amplification when solving a system of linear equations. In robotics, the condition number was first used by [3] in the optimization of a mechanical two-phalanx finger. In order to enable the calculation of this index in the presence of a dimensionally heterogeneous Jacobian, [4] defined the *homogeneous Jacobian* matrix by introducing the *characteristic length*. The concept was also used for the design of mechanisms, as reported by Wang et al. [5] and Mermertas [6]. Nawratil [7] expanded the concept of condition number for an operation ellipsoid instead of an operation point[4] (OP). Manipulability is the absolute value of the determinant of the Jacobian matrix, first used by Paul and Stevenson [8] to design a wrist. A good manipulability index was claimed to indicate a point in the workspace "far" from singularities. As Kim and Khosla [9] argued, manipulability being the product of the Jacobian singular values, it is both scale- and order- (number of joints) dependent, which prevents its use for the comparison of manipulators of different sizes and of different numbers of joints. Nevertheless, comparing two robots in the second category would be unfair, regardless of the index used for comparison.

The manipulator velocity ratio, also known as the velocity-transmission ratio, and the manipulator mechanical advantage, also known as the force-transmission ratio, were proposed as performance indices by Dubey and Luh [10]. The former measures the robot ability to move in a given direction, the latter represents the robot ability to balance a given wrench—consisting of the applied force and moment vectors. In the same vein, Legnani et al. [11] proposed the concept of *point of isotropy* to design new manipulators with velocity, force and stiffness isotropy, and to make existing robots isotropic by changing the dimensions of the EE.

As well, the quality of force transmission is one of the important criteria in the design of mechanisms. For four-link planar mechanisms, the transmission angle was introduced by Alt [12,13]. An extensive literature survey of the transmission angle and its application to planar and spatial linkages was conducted by Balli and Chand [14]. When the transmission angle is 90°, the force applied to the output link is aligned with its motion, thus minimizing the force transmitted to the machine frame. The concept of *virtual coefficient* between two screws was introduced by Ball [15]. Yuan et al. [16] defined the *transmission factor* for spatial mechanisms based on the virtual coefficient between the twist and the wrench screws. Later, [17] introduced the *transmission index* (TI) by dividing Yuan et al.'s transmission factor by a maximum virtual coefficient defined at a *characteristic point*. As Chen and Angeles [18] argued, the maximum virtual coefficient is not valid when one of the screw pitches is infinite or the screws are parallel; to cope with these cases, the generalized TI (GTI) was introduced in the foregoing reference. The GTI is the virtual work normalized by means of a global maximum, which is obtained at a point called the *application point*.

For a mechanism, the TI is the relation between the wrench applied onto and the twist of the output link; indeed, the reaction wrench applied onto a floating link by its neighbors, free of external loads, is reciprocal to the link twist. As Holte and Chase [19] and Lin and Chang [20] explained, the TI does not take into account the influence of external loads; it is purely posture- and link-geometry dependent. Holte and Chase [19] proposed a force transmission index, called *joint-force index*, as the ratio of the maximum static force at any joint to the external load. The latter could be the pure force or the torque but not a combination of both. Furthermore, when the external load is a torque, this index has units of reciprocal length; thus, the comparison between two mechanisms of different sizes is not possible with this index. Lin and Chang [20] proposed the *force transmissivity index* as a combination of the ratio of the effective power to the maximum power and the mechanical advantage. This index is bounded from below by zero, and unbounded from above, while becoming undefined when the mechanism approaches a deadpoint.

In Section 2, a new index is defined by taking into consideration the torque applied at the joints; the index is thus task-dependent, besides being posture-dependent. The new index, termed here *robot transmission ratio* (RTR), is obtained as the absolute value of the cosine of the angle between the joint-torque and the joint-rate vectors, for n-revolute serial robots; thus, the index is dimensionless and ranges from 0 to 1. Notice that for the purpose of this paper, trajectory planning, these vectors, defined in the joint space, are generally unknown. Therefore, the foregoing vectors are replaced by the wrench and twist arrays of the end-effector (EE) via the Jacobian matrix, the RTR being calculated in the Cartesian space. The twist and the wrench of the EE are known, at least partially, in machining operations, as the trajectory and the wrench produced by the cutting force applied to the end-effector (EE) are the data of the problem. As well, it is noteworthy that most industrial serial robots have only revolute joints; thus, the RTR, as proposed here, is not applicable to robots with prismatic pairs.[5]

In Section 3, the RTR is simplified for a six-revolute industrial robot of the decoupled type, i.e., with its last three axes concurring at a common point C, the center of the *spherical wrist* formed by these axes. Further, it is assumed that the angular velocity of the end-effector is negligible with respect to its translational velocity *for the usual dimensions of tool, tool-holder and robot*. Moreover, the cutting torque applied to the tool is neglected by virtue of a) the relatively small depth of cut, since robots are used in finishing operations, b) the relatively small value of the moment of the cutting force, when the spindle turns at speeds of the order of 10,000 rpm, and c) its small lever arm with respect to the axis of the spindle. These assumptions make the calculation of the RTR possible with no need of knowledge of all six components of the twist and of the wrench involved. The simplified RTR is shown to be a function of both the task (velocity and force direction) and the robot posture (Jacobian matrix), but not of the velocity and force *amplitudes*, using a term proper of screw theory [15].

[4] The OP, also known as the tool-center point (TCP), is defined as the point of the EE at which the task is specified.

[5] The index could still be calculated if a certain characteristic length were introduced to render the Jacobian dimensionally homogenous in light of the disparate type of the robot joints. For the sake of brevity, this issue is not pursued here.