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An investigation on stiffness of a 3-PSP spatial parallel mechanism with flexible moving platform using invariant form

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ABSTRACT

In this paper, the stiffness of a 3-PSP spatial parallel manipulator is investigated. Unlike traditional stiffness analysis, the moving platform is assumed to be flexible. Two analytical methods are used in finding the robot stiffness. In the first method, robot is modeled as lumped system and principle of virtual work is used. In the second method, the robot is modeled as a distributed system and strain energy of robot main components as well as Castigliano's theorem are used. Force analysis is also presented and reaction forces at the joints as well as internal forces/moments are obtained. For each of the main robot components, a matrix called Wrench Compliant Module Jacobian, WCMJ, is introduced. These matrices will allow mapping the applied external wrench on the moving platform to corresponding reaction forces for the corresponding compliant module. All analysis is presented using invariant form. To evaluate accuracy of the two methods, finite element analysis is used. Finally, using the distributed method, maximum and minimum eigenvalues of the stiffness matrix are obtained and values of kinematic stiffness index are presented.

1. Introduction

Application of parallel robots in industry continues to increase [1]. Some of these applications are simulators, machine tools, cutting and welding machines as well as CNC machines [2–5]. High precision and stiffness as well as good dynamic efficiency of parallel robots give them the capability to be used as CNC machines [5]. Earlier parallel robots usually have six degrees of freedom [6–8]. However, today, with increased application of parallel robots, robots with fewer numbers of degrees of freedom are needed [9,10]. These robots, in addition to having most of the capabilities of the parallel robots, can be made with less cost [11–13]. The economical factor of parallel robots with lower degrees of freedom has expanded their use in manufacturing processes [14,15]. In designing of parallel robots various criteria such as workspace, maximum capacity of load carrying, stiffness and KSI (kinematic stiffness index) should be investigated [16–20]. When parallel robots are used as machine tool, stiffness is considered one of the most important design parameters [4,15,21]. In fact, in parallel robots, accuracy has a direct relationship with stiffness of the robot. Accuracy and stiffness are two important parameters considered when designing machine tools [13,22]. Therefore, it is natural to consider use of inherently stiff parallel robots in machine tools and CNC machines. To study the stiffness of robots, two methods may be used to find the stiffness matrix of robot. The first method uses theoretical formulation while the second method uses actual experiments performed on robot [23]. Rezaei and Akbarzadeh [1] studied stiffness of a spatial parallel robot by considering flexibility effect of the moving platform using a distributed approach. Also, Enferadi and Akbarzadeh [24] investigated the stiffness of a spherical parallel robot by calculating strain energy of each component of the robot. However, in most studies, stiffness model of the parallel robots is considered as lumped. Li and Xu [22] derived the stiffness matrix of a 3-PUU PKM based on an alternative approach considering actuations and constraints. Kim and his coworkers

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investigated the stiffness analysis of a 3-DOF parallel robot with one constraining leg, which takes into account the elastic deformations of joints and links. To obtained stiffness matrix, overall Jacobian matrix and principle of virtual work are used [25].

The stiffness analysis of general 6-DOF parallel manipulators has been extensively reported, specifically, Stewart–Gough platform [26]. Gosselin used Jacobian matrix to study the stiffness of Stewart platform and the mapping between the driving force and the platform deformation [27]. To evaluate the robot stiffness variations throughout the workspace and to obtain the effect of altering the kinematics parameters in the structure, the eigenvalues of the stiffness matrix and KSI criteria are used [12,19,21,28]. In [20] a general and semi analytical approach for formulation of the stiffness matrix of parallel robot and its comparison with FEA is presented.

The purpose of stiffness analysis is to obtain its related stiffness matrix. Stiffness matrix relates 6 dimensional vector of small displacement for the end-effector and its corresponding 6 dimensional vector of applied static forces/torques (wrench). In robots, stiffness has a direct relationship with precision and maximum load carrying capacity [24]. It can be demonstrated that stiffness of a robot is limited between maximum and minimum eigenvalues of its stiffness matrix [12,22,28]. One method used to evaluate the stiffness is finding the maximum and minimum eigenvalues of the stiffness matrix and using KSI criterion [29]. Since the stiffness is a 6×6 matrix, 6 eigenvalues can be found. The stiffness of robot in its workspace can be evaluated by finding the maximum and minimum of its 6 eigenvalues throughout this space. For machine tool applications, the robot physical parameters should be designed so that the minimum values of the stiffness matrix, in its workspace, is greater than a desired value [3,22,19,28]. This will results in a desired accuracy for the machine. Therefore, determining low and high limits for stiffness of a robot is considered an essential part of a machine design [12,30].

In this paper, two analytical methods for solving the stiffness of 3-PSP parallel robot are presented. The presented methods are general and can be applied to most parallel robots. In the first method, stiffness of the robot is modeled as lumped and solved using principle of virtual work. In this method all flexible components such as, Linear rods and motors are modeled using linear springs. Jacobian analysis is first performed to find the relationship between displacement of the end-effector and the corresponding displacement of the actuators. Next, using principle of virtual work, the relationship between deformations of the end-effector and corresponding external wrench on the robot tip is obtained [13,22,25]. In the second method presented in this paper, unlike the first, stiffness of the robot is modeled as distributed system. This method is based on Castigliano's theorem and calculation of strain energy of the robot components. Traditional methods used for calculation of the robot stiffness, are based on modeling of stiffness as lumped. There are many limitations and assumptions used for simplification of the lumped model. However, when the robot is modeled as distributed system, there is no need to use any of the simplifying assumptions. Therefore, this method will be more accurate in modeling the stiffness. Furthermore, this method allows us to model the star shaped moving platform as a flexible body and include the effect of bending in all components of the robot. The results of the two methods, virtual work and Castigliano's, are further compared with results from a commercial finite element analysis software.

This paper is organized as follows: In Section 2, structure of a spatial 3-PSP parallel robot is introduced and solution to inverse kinematics of the robot is presented. In Section 3, a lumped stiffness model is presented using principle of virtual work. Next, the second method is presented for solving robot stiffness based on Castigliano's theorem. Calculation of the strain energy for the robot components assuming continuous model for the robot are presented. In Sections 4 and 5, the results of the two previous models are each compared with results from the FEA model. The method with higher accuracy is used for the subsequent analysis. In Section 6, the more accurate method is selected and the robot stiffness, using maximum and minimum eigenvalues of stiffness matrix, in its workspace is evaluated. The kinematic stiffness index (KSI) is also calculated for several sections of the workspace.

2. Structural description and inverse kinematics analysis

In this paper, a special type of 3-PSP parallel robot is investigated. The solid and physical models of a 3-PSP parallel manipulator are illustrated in Fig. 1(a) and (b). This robot is a fully parallel mechanism with three degree of freedom. This robot is composed of a moving platform which is shaped like a star and two fixed platforms. Selected tools may be placed in the center of the moving platform also referred to as moving star (MS). The moving star and the fixed platforms are connected together with three parallel legs with identical serial kinematic chains. Each of the three legs, consists of an active prismatic joint (P-joint), actuated by a Linear rod (LR), and a passive spherical joint (S-joint), followed by a second passive prismatic joint. Therefore, the MS is attached to the base by three identical serial PSP linkages. The three independent DOFs for the robot may be selected among the six possible degrees of freedoms (x, y, z, θ, φ andλ). In the present paper, two rotational and one translational variables θ, φ and z are selected as inputs for the inverse kinematics problem (see [2] for more details). Fig. 1(c) shows geometry for one of the three kinematic chains.

The vectors and reference frames are also described in this figure. A fixed coordinate frame B{x, y, z} is arbitrarily embedded in the top fixed platform and attached to the center point O of fixed triangle $\Delta A_1A_2A_3$. Likewise a moving coordinate frame T{u, v, w} is attached to the tool, at point T. In this paper, vectors referenced in fixed base coordinate frame {B} are denoted by $^B\mathbf{v}$, while vectors referenced in moving coordinate frame {T} are denoted by $^T\mathbf{v}$. The three spherical joints are denoted by S_i. Three position vectors $^B\mathbf{q}_i$, defined in {B}, connect corners of the fixed triangle, A_i, to the center of the spherical joints, S_i. Position of the end-effector (point T) with respect to {B} is given by vector ^BT. Three additional position vectors, $^B\mathbf{a}_i$ locate corners of the fixed base, A_i, in {B}. The position vector $^T\mathbf{b}_i$, connects the end-effector, point T, to the ith spherical joint, S_i, and is defined in {T}.

Consider Fig. 1(c). Three closed vector-loop equations can be written as,

$$^B\mathbf{a}_i + {}^B\mathbf{q}_i = {}^B_T\mathbf{R}\,{}^T\mathbf{b}_i + {}^B\mathbf{T} \text{ for } i = 1, 2, 3 \quad (1)$$