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Analyzing the topological, electrical and reliability characteristics of a power transmission system for identifying its critical elements

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ABSTRACT

The subject of this paper is the analysis of an electrical transmission system with the objective of identifying its most critical elements with respect to failures and attacks. The methodological approach undertaken is based on graph-theoretical (topological) network analysis. Four different perspectives of analysis are considered within the formalism of weighed networks, adding to the purely topological analysis of the system, the reliability and electrical characteristics of its components. In each phase of the analysis: i) a graph-theoretical representation is offered to highlight the structure of the most important system connections according to the particular characteristics examined (topological, reliability, electrical or electrical-reliability), ii) the classical degree index of a network node is extended to account for the different characteristics considered. The application of these concepts of analysis to an electrical transmission system of literature confirms the importance of different perspectives of analysis on such a critical infrastructure.

1. Introduction

Engineered critical infrastructures are the systems of interest in this work. The motivation is that they provide the continuous flow of essential goods (e.g. energy, water and data) and services (e.g. banking, health care and transportation) which the welfare and security of our nations rely on. These critical infrastructures are subject to a set of multiple hazards and threats which must be identified and analyzed for optimal protection.

Among the engineered critical infrastructures, the focus of this work is on the electrical transmission system and its analysis to identify the importance of the individual elements. The motivation is that the infrastructure for electrical transmission of most of the world's countries is aging and failing, with funding often not sufficient to repair or replace it; in this situation, there is a growing demand for a rational, risk-based approach to its operation and maintenance.

A number of recent studies have addressed the assessment of vulnerability in electric power systems, by graph-theoretical topological investigations as in [1–5], physics-based models as in [6–9], agent-based modeling as in [10]. These studies all refer to the transmission system and are based on different conceptualizations of vulnerability [11]. Also, more sophisticated techniques such as polyhedral dynamics [12] and artificial intelligence-based search methods [13,14] have been proposed to find critical elements and define vulnerability indices.

The topological approach to vulnerability analysis is quite popular because in spite of its relative simplicity it offers the capability of identifying elements of structural vulnerability, i.e. network edges and nodes whose failure can induce a severe structural damage to the network through the physical disconnection of its parts. However, the methods based on such approach are limited from the point of view of the physical analysis of the electrical transmission systems, which the graphical networks represent; the limitations come from the fact that the analysis focuses only on the topological features of the network, thus neglecting its physical characteristics [15–17]. In this respect, it is important to verify the extent of these limitations and possibly overcome them by complementation with more detailed physical analyses on critical parts of the network [18,19].

In this paper, the formalism of weighed networks is exploited to provide different graph-theoretical representations and analyses of a power transmission system. The aim is to contribute to reducing the gap between the highly conceptualized (and abstract) analyses based purely on considerations of the system topology and the highly detailed (and computationally demanding) simulations of system behavior, in order to render the overall vulnerability assessment more feasible and robust. The "weights"

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appended to the network elements are intended to capture relevant electrical and reliability properties of the system, so as to overcome the classical simplifying but unrealistic assumption that electrical flow occurs along the shortest and failure-free paths of connections.

The rest of the paper is organized as follows: Section 2 describes the theoretical basis for the proposed perspectives of analysis. In Section 3, the work is presented from a practical point of view with reference to the popular IEEE RTS 96 system [20]. Section 4 contains a critical comparison of the four perspectives of analysis with an outlook on other perspectives. Conclusions are drawn in Section 5.

2. Different perspectives of analysis

In the present study, the system is modeled as a stochastic, weighted, undirected, connected network in which each electric bus is transposed into a node, linked by edges representing the overhead lines connecting consecutive buses. In this respect, this representation focuses on the actual topological structure of the power transmission network.

Mathematically, the topological structure of the network can be represented as an undirected graph $G(V,E)$ where V represents the set of vertexes (or nodes, or components) ($N=\dim(V)$ is the number of nodes) and E represents the set of edges (i,j) ($K=\dim(E)$ is the number of edges). The connections are specified in an $N\times N$ adjacency matrix $\{a_{ij}\}$ whose entries are 1 if there is an edge joining node i to node j and 0 otherwise.

2.1. Graph-theoretical topological analysis

The transmission network is first analyzed from a purely topological point of view. In the topological representation, no specification of the physical length of the edges is given. Each link is considered having a length equal to one and thus the distance between two nodes i and j is represented solely by the number of edges traveled in the path from i to j. On the basis of $\{a_{ij}\}$, it is possible to compute the matrix of the shortest path lengths $\{d_{ij}\}$ whose generic entry d_{ij} is the number of edges making up the shortest path linking i and j in the network. The fact that G is assumed to be connected implies that d_{ij} is positive and finite $\forall i\neq j$ and that there are $N(N-1)/2$ distinct shortest paths among the N nodes.

From the matrix of shortest path lengths $\{d_{ij}\}$, a new matrix $\{s_{ij}\}$ can be computed by considering connected only a number of K_S links with smallest values of shortest path lengths; the generic element s_{ij} is equal to 1 if the shortest distance connecting i and j is one of the K_S smallest values and 0 otherwise.

A synthetic indicator of the topological structure of a complex network is the distribution of the degree (or connectivity) k_i of its nodes $i=1,2,...,N$, the degree being defined as the number of edges incident to the node [21]:

$$k_i=\sum_{j\in N}a_{ij}, i=1,2,\ldots,N \tag{1}$$

Intuitively, the degree of a node measures its influence in the graph with respect to the size of its immediate environment.

2.2. Reliability analysis

While some studies witnessed a reasonable association between the topology of the power grids and the robustness and stability of the power transmission systems [1,22,23], the relationship between network structure and system reliability is also of relevance. In this respect, the formalism of weighted networks [22] can be undertaken to account for the reliability properties of the transmission network system. More precisely, a reliability matrix $\{p_{ij}\}$ can be introduced to describe the network reliability properties at a local level [24]; the generic element p_{ij} represents the probability of successful transmission along the edge that connects node i and j.

Since the graph is not fully connected, the reliability matrix tends to be sparse ($p_{ij}=0$ for all pairs of nodes i and j that do not share a direct physical connection). In order to obtain a non-sparse matrix containing the complete information on the reliability of connection between any two pairs of nodes, the reliability $p_{\gamma_{ij}}$ of connection between i and j through any connecting path γ_{ij} is computed by a method based on a combination of cellular automata (CA) and Monte Carlo (MC) sampling [25]. In this method, the reliability $p_{\gamma_{ij}}$, i.e. the probability of a successful connection from i to j, is computed by MC—sampling a large number M of random realizations (MC trials) of the states of the connecting arcs and by CA-computing, for each realization, if a path from i to j exists; the ratio of the number of successful γ_{ij} paths over the total number of realizations computed gives the connection reliability from node i to node j.

As indicator of the importance of the nodes from the reliability point of view, a reliability degree can be introduced as:

$$k_i^r=\sum_{j\in N}p_{\gamma_{ij}},\quad i=1,2,\ldots,N \tag{2}$$

From the reliability analysis, a matrix $\{s_{ij}^r\}$ is computed on the basis of the K_S most reliable paths. The matrix element s_{ij}^r is equal to 1 if the connection from node i to j is one of the K_S most reliable connections, and 0 otherwise. $\{s_{ij}^r\}$ can be thought of as the reliability equivalent of the topological adjacency matrix $\{a_{ij}\}$.

2.3. Electrical analysis

As mentioned in the Introduction, in the case of electrical transmission networks of interest here, the existing literature on vulnerability analysis largely takes a topological approach to identify the critical components in the network [1,26,5]. Even though such analyses are capable of identifying elements of structural vulnerability, they are limited from the point of view of the physical analysis of the electrical transmission system, which the networks represent. These limitations are all related to the fact that the analysis performed focuses only on the topological features of the network, thus neglecting its physical characteristics; this is not realistic for electrical transmission networks in which: i) the "electrical" length of a path differs from the topological length, depending on the difficulty (resistance) of transmission, ii) the electrical power is not necessarily routed through the shortest paths, rather, the transmission of power is determined by physical rules, e.g. Kirchoff's laws, nodal voltages, etc.

To practically overcome these limitations, an electrical connectivity metric was introduced within the weighed network formalism to capture the properties of node centrality, relative to metrics based on node-edge connectivity [16].

The electrical characteristics of the individual network elements and their interconnection relationships can be expressed in terms of the bus admittance matrix, Y^{bus}. Inverting the sparse bus admittance matrix ($Y_{ij}^{bus}=0$ for all pairs of nodes i and j that do not share a direct physical connection) that incorporates Kirchoff's laws, a non-sparse matrix, known as impedance matrix, can be obtained.

The matrix of electrical distances is then given by the magnitude $\{m_{ij}\}$ of the entries of the matrix Z^{bus}. Admittance is a complete expression of the extent to which a circuit allows a current to flow; as the absolute value of the complex admittance