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A computational investigation on the reducing lateral vibration of rotors with rolling-element bearings passing through critical speeds by means of tuning the stiffness of the system supports

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ABSTRACT

Since there are manufacturing and assembling inaccuracies rotors are always slightly imbalanced. Their rapid angular acceleration leads to an increase in the magnitude of the tangential component of the imbalance forces. The slow acceleration enables the development of an almost steady state vibration. In both cases the amplitude of their oscillation is high. The frequency tuning achieved by changing the stiffness of the rotor supports represents one approach to its attenuation. Stiffness decreases shortly before reaching the critical speed, and after crossover, the stiffness increases again. As almost all publications and computer procedures from the field of rotor dynamics deal only with the case when the rotor turns at a constant angular speed, derivation of the equations of motion of the rotor rotating at variable velocities and investigation of the operating conditions and individual parameters of the supports on the vibration of the rotor passing critical speeds represent the fully new contribution of this paper. The critical revolutions are determined from the Campbell diagram. To solve the equations of motion the Newmark method has been proposed and tested.

1. Introduction

The rotor systems consist of two principal components: a rotor and a stationary part. Because of their high loading capacity and high stiffness rolling-element bearings are often used for their coupling.

The rotors of rotating machines are always slightly imbalanced due to manufacturing and assembling inaccuracies. Especially if they run over the critical speeds, the imbalance produces a vibration of a large amplitude. Rapid angular acceleration leads to an increase in the magnitude of the tangential component of the imbalance forces. Slow acceleration enables the development of an almost steady state vibration. In both cases the amplitudes of the induced oscillations are high.

The principal aim of this article is to contribute to the development of computational procedures for an efficient analysis of the lateral vibration of rotors supported by rolling-element bearings. Particular attention is paid to the case when the rotor increases or decreases the speed of its rotation when passing critical revolutions and to the attenuation of its vibration by frequency tuning. This manipulation requires a change in the stiffness of the rotor supports when it rotates at a speed close to its critical revolutions. The experience showed that efficiency of this manipulation depended not only on the magnitude and rate of the stiffness change but also on the moment of time when the tuning started before reaching the critical rotational speeds. From a technological point of view this enables the use of a motor of a lower rated power. There are several design possibilities to change the stiffness of the supports. A mechanical one based on the change of the length of an elastic spring is described in [1]. The controlled change in the

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stiffness of a rotor by a bearing movable in the axial direction is reported in [2]. For investigation of the elimination of the rotor unbalance the authors used only a simple rotor of a Jeffcott type. Another possibility on how to change the stiffness of the structure supports is represented by piezoelectric devices. For example, the stiffness of a lead-zirconate-titanate piezoceramic actuator with short-circuited electrodes on its faces is smaller by about 20–40% than in the case when the opposite faces of the actuator are not electrically connected. Further possibility is the usage of smart memory metals [3]. The spring elements supporting the rotor are made of shape memory alloys and the change in their stiffness is accomplished by heating or cooling them.

To analyze this problem and to propose the optimal speed of passing the critical revolutions a computer modelling method can be used. Unfortunately, many publications and computer procedures from the field of rotor dynamics are related only to the case when the rotor turns at a constant angular speed. Moreover, if the discs or rotors turning at a variable speed are investigated, the forms of the equations of motion derived by different authors slightly differ and the differences depend on the method used for their derivation. The equations of motion of a symmetric rotating disc having four degrees of freedom have been derived in [4]. Using the assumption of small displacements and rotations, the authors simplified Euler's dynamical equations by neglecting small terms of the second and higher orders but the resulting relationships are referred again to a disc rotating only at a constant angular speed. In [5] the equations of motion for a Stodola rotor are derived by means of the impulse theorems. Even if they are related only to constant speed of the rotor rotation, they can be easily extended for the case when the rotor turns with variable revolutions. Then the terms of inertia moment for rotor spin motion would occur in both moment equations. Such equations of a flexibly supported disc performing a spherical movement with variable speed are used in [6–9]. The system has two degrees of freedom and therefore its vibration is governed by two equations. In [10] the motion equations of a disc and of a shaft element are derived. For their derivation the Lagrange's equations of the second kind were used. The resulting equations of motion are asymmetric because only one moment equation contains the inertia term of the spin motion of the disc or of the shaft element. Moreover, if the rotations about the axes of the reference frame used by the authors were performed in reverse order, the inertia term would appear in the other moment equation which because of validity of the principle of superposition is not acceptable. The equations of motion referred to a rigid disc rotating at variable speed and derived by means of the Lagrange equations of the second kind which can be found also in [11]. Both resulting moment equations of motion do not contain any inertia terms corresponding to the spin motion of the disc. This form of the equations of motion was also used by some other authors, e.g. [12]. As evident from this survey, it is desirable to deal with the derivation of the governing equations of flexible rotors turning with variable speed in more details and to discuss the mentioned differences.

In the mathematical models the shaft is usually represented by a beam-like body that is discretized into finite elements and the discs are considered to be thin and rigid. Derivation of the mass, stiffness, and gyroscopic matrices of the disc and of the shaft elements has been performed by Nelson and McVaugh [13] but now it can also be found in a number of further publications, e.g. in [14]. Implementation of the internal damping into the shaft element was done by Zorzi and Nelson in [15]. However, all these equations of motion are valid only if the rotor rotates at a constant angular speed. The rolling-element bearings have almost no damping and are distinguished by their high stiffness. They are implemented into mathematical models by means of linear or non-linear isotropic or orthotropic springs and the damping elements that produce forces in two mutually perpendicular directions. A determination of the stiffness and damping parameters of ball and roller bearings with no pre-stress and zero clearances between the rolling elements and the races is given in [16].

The problem of attenuation of rotors passing critical speeds by tuning their support stiffness has been studied since approximately the 60s in the 20th century. Irretier and Leul [17] investigated a single-degree-of-freedom system with no internal damping and gyroscopic effects. The angular acceleration of the rotor was constant and the change of the system's natural frequency was linear with respect to time. Nagaya et al. [3] studied the undamped multi stepped rotor with one disc. The gyroscopic effect was taken into account and the shaft deflection was described by an analytical function. Ballo and Chmurny [18–20] analyzed the influence of the magnitude and rate of decrease of the shaft support stiffness and of the moment of time when this manipulation should start on reducing the rotor vibration amplitude. Their mathematical model was very simple (no damping, no gyroscopes effects, two degrees of freedom, constant cross section of the shaft, no discs) and they used a small parameter method. Wauer and Suherman [21] focused their work on the study of the influence of the change in the support stiffness on the rotor lateral vibration and on the driving moment needed for crossing the resonance speed. Their rotor was represented by a slender bar of a constant circular cross section. Millsaps and Reed [22] dealt with lateral vibration of a Laval rotor passing the critical speeds with constant acceleration or deceleration. They analyzed the effect of the rotor angular acceleration on amplitude of the induced vibration and on fluctuation of the kinetic energy related to the whirl and spin motions of the rotor.

The principal contribution of the presented article consists in deriving the equations of motion for a flexible rotor with rigid discs rotating at a variable angular velocity utilizing the impulse theorems for this purpose and the principle of the virtual work and development of an efficient procedure for investigation of the attenuation of the amplitude of rotor vibration passing the critical speed by a change in the stiffness of the shaft supports. The critical revolutions are determined from the Campbell diagram, explained in [5,16,23]. For solving the motion equations of the whole rotor system the Newmark method has been chosen. Applicability of the developed approach was tested by means of computer simulations.

2. The equation of motion of a rigid disc

In the proposed mathematical model it is assumed that (i) the disc is a thin, rigid, axisymmetric body, (ii) the middle plane of the disc is perpendicular to the centre line of the shaft, (iii) the force transmission between the disc and the shaft is accomplished