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Elasto-geometrical modeling and calibration of redundantly actuated PKMs

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ABSTRACT

Redundantly actuated parallel kinematic machines (PKMs) offer a number of advantages compared to classical non-redundant PKMs. Particularly, they show a better stiffness thanks to singularity avoidance and they have an improved repeatability due to a better behavior against backlashes. The main problem with the calibration of these machines is that the redundancy leads to some mechanical strains in their structure. This makes it difficult to identify the geometrical errors of their structure without taking into account the effects of the elastic deformations. The main originality of this work is to propose an efficient elasto-geometrical and calibration method that allows the identification of both the geometrical and stiffness parameters of redundantly actuated parallel mechanisms with slender links. The first part of the paper explains the proposed method through its application on a simple redundant planar mechanism. The second part deals with its experimental application to the redundant Scissors Kinematics machine.

1. Introduction

Redundantly actuated parallel kinematic machines (PKMs) have recently attracted interest of researchers because they allow the reduction of some drawbacks of classical, non-redundant PKMs [1]. The presence of one or more redundant actuated chains in the structure allows the avoidance of mechanism singularities [2,3] and the reduction of joint backlash effects using a control on the internal force directions [4–6]. Those redundant chains can also be used to perform the autonomous calibration of these mechanisms [7,8].

Whereas for classical PKMs am insufficient knowledge of the mechanism geometrical properties, such as link length or joint position/orientation, leads exclusively to Cartesian position inaccuracies at the tool center point (TCP) [9], in the case of redundant PKMs, such errors also lead to internal constraints. These mechanical strains in the structure can in turn cause early part wearing and loss of energy in the actuators [5,10]. These internal constraints make it difficult, not to say impossible, to identify the errors of the geometrical parameters involved in the control model without taking into account the resulting structure elastic deformations. The purpose of this paper is to provide a computation method of the platform situation (position/orientation) of redundantly actuated PKMs and to show how it can be involved in their calibration. By using this method, one can improve the calibration quality of redundant PKMs by carrying out backlash-free measurements while taking into the effect of the internal constraints used to reduce this backlash.

This paper is organized as follows. First, the proposed methodology that is used to derive the elasto-geometrical models of parallel mechanisms with one or more actuated redundant chains is described for planar mechanisms. The development of the method is then illustrated on the Redundant Triglide, a simple redundant planar mechanism. Explanations are then given

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to show how this modeling method can be easily extended to three dimensional redundant parallel mechanisms. Then, the calibration strategy that has been used to perform the geometrical and stiffness parameter identification of the obtained models is explained. Sensitivity and observability analyses as well as the results of calibration simulations show the efficiency of the proposed approach. The last section deals with the experimental application of the method for the elasto-geometrical modeling and calibration of the redundant Scissors Kinematics machine developed at the Fraunhofer Institute of Machine Tools and Forming Technology IWU in Chemnitz, Germany.

2. Elasto-geometrical modeling of redundant PKMs

2.1. Method description

The high dynamics of PKMs suppose low moving masses, i.e., slender elements and light joints [11,12], which are then subject to elastic deformations. For calibration purposes, these elastic deformations that depend on the PKM configuration [13] have to be calculated in order to be compensated. For this purpose, an analytical finite-element modeling using beam elements is proposed to describe redundant PKMs. This approach enables a reduction in the calculation times as well as the number of parameters in comparison to a CAD finite-element method with numerous surface or volumic elements. This makes it compatible with calibration issues [14]. However, the calculation of the platform situation for redundant PKMs cannot be achieved in a similar manner as for non-redundant PKMs because it involves an over-determined equation system [15]. The number of loop-closure equations is greater than the platform's degree of freedom. Some authors proposed some purely geometrical methods [16] and some methods based on lumped models [5,17]. However, the analytical finite-element method allows a more accurate calculation of the platform situation because all possible deformation effects are taken into account [18,19]. Thus, the final aim of the modeling is to obtain the platform position/orientation $\boldsymbol{X} = (\boldsymbol{P}^{\mathrm{T}}\, \boldsymbol{\phi}^{\mathrm{T}})^{\mathrm{T}}$ under actuation redundancy.

The modeling method is based on the following steps:

(1) Calculation of the platform position/orientation $\boldsymbol{X}_{\mathrm{nr}} = \left(\boldsymbol{P}_{\mathrm{nr}}^{\mathrm{T}}\, \boldsymbol{\phi}_{\mathrm{nr}}^{\mathrm{T}}\right)^{\mathrm{T}}$ thanks to the forward geometrical model (fgm) of a non-redundant substructure of the redundant mechanism: $\boldsymbol{X}_{\mathrm{nr}} = \mathrm{fgm}(\boldsymbol{q}_{\mathrm{nr}}, \xi)$, where $\boldsymbol{q}_{\mathrm{nr}}$ is the vector of actuator positions of the non-redundant subsystem and ξ, the vector of the geometrical and stiffness parameters.

(2) Calculation of the platform displacements induced by the structure elastic deformations due to its own weight and the applied external forces. This is done through a forward elastic model (fem) $\Delta\boldsymbol{X}_{\mathrm{e}} = \mathrm{fem}(\boldsymbol{q}, \xi, \boldsymbol{F})$, where $\boldsymbol{q}$ is the vector of all actuator positions and $\boldsymbol{F}$ the wrench including the external forces acting on the structure and the structure's own weight.

(3) Calculation of the final platform position/orientation through the resulting forward elasto-geometrical model fegm : $\boldsymbol{X} = \mathrm{fegm}\ (\boldsymbol{q}, \xi, \boldsymbol{F}) = \boldsymbol{X}_{\mathrm{nr}} + \Delta\boldsymbol{X}_{\mathrm{e}}$.

In order to derive the forward elastic model (fem) involved in step (2), the following finite-element approach is used:

- Determination of the stiffness matrices of all links and joints of the mechanism within a local frame attached to each link and joint.
- Calculation of all stiffness matrices into the global reference frame of the structure.
- Mapping and assembly of all the resulting stiffness matrices to derive the global stiffness matrix of the structure.
- Calculation with the global stiffness matrix of the structure of the displacement of all nodes and in particular of the node that corresponds to the TCP.

In next section, the proposed method is detailed for redundant planar parallel mechanism and then expanded for three dimensional structures.

2.2. Elasto-geometrical modeling of redundant planar mechanisms

In this section the description of the method is presented for planar mechanisms. For this purpose, the values of the geometrical parameters are considered as nominal, that is to say without any errors. As a result, the position and orientation of the platform associated frame is identical and can be calculated with any non-redundant subsystems of the structure. A non-redundant subsystem is defined here as a part of the mechanism that contains as many actuated chains as the platform's degree of freedom and that constitutes a viable mechanism.

2.2.1. Modeling of the structure links

The slender links of the planar structures are considered as rods that can be described by planar 2-node beams. For each beam, a local reference frame $\Re_{ij}$ is defined in such a way that its origin is at node i, its $\boldsymbol{x}$-axis goes through nodes i and j and its $\boldsymbol{z}$-axis is the same as the $\boldsymbol{z}$-axis of the global frame that is denoted $\Re_g$ (Fig. 1). The stiffness of each two-node beam is first expressed in the beam local reference frame $\Re_{ij}$ as [20]: