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Multi-state systems with selective propagated failures and imperfect individual and group protections

Gregory Levitin [a,b,*], Liudong Xing [a,c], Hanoch Ben-Haim [b], Yuanshun Dai [a]

[a] Collaborative Autonomic Computing Laboratory, School of Computer Science, University of Electronic Science and Technology of China, China
[b] The Israel Electric Corporation, P.O. Box 10, Haifa 31000, Israel
[c] Electrical and Computer Engineering Department, University of Massachusetts, Dartmouth, MA 02747, USA

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ABSTRACT

The paper presents an algorithm for evaluating performance distribution of complex series–parallel multi-state systems with propagated failures and imperfect protections. The failure propagation can have a selective effect, which means that the failures originated from different system elements can cause failures of different subsets of elements. Individual elements or some disjoint groups of elements can be protected from propagation of failures originated outside the group. The protections can fail with given probabilities. The suggested algorithm is based on the universal generating function approach and a generalized reliability block diagram method. The performance distribution evaluation procedure is repeated for each combination of propagated failures and protection failures. Both an analytical example and a numerical example are provided to illustrate the suggested algorithm.

1. Introduction

Considerable research efforts have been devoted to modeling common cause failure (CCF) distributions and estimating the effect of CCF on system reliability or availability (see, e.g., [1–10,23–26]). CCFs are one type of dependent failures that increase joint-failure probabilities, thereby reducing the reliability of technical systems. In particular, CCFs are multiple components failures due to a common cause. The origin of a common cause event can be outside the system it affects, for example, a lightning event that causes outages of unprotected electronic equipment. CCFs can also originate from the system elements themselves, causing other elements to fail, for example, voltage surges caused by inappropriate switching in power systems lead to failure propagation. In this latter case, CCFs are also known as propagated failures (PFs). Depending on the set of elements affected by the PFs, the global and selective effect PFs can be distinguished. A PF with global effect takes place when the PF originated from any element of a subsystem causes the failure of the entire subsystem. It can happen in systems with imperfect fault detection/coverage mechanism [11–20] or systems without any fault coverage mechanism [30]. A PF with selective effect takes place when the PF originated from different system elements cause failures of different (possibly overlapping) subsets of the system elements. The PFs with selective effect have been studied in [27,28] for binary-state systems and in [29] for multi-state systems that can function at different states characterized by different levels of performance.

The propagated failures originated from system elements can be treated as outer impacts to the rest of system elements. By protecting individual system elements and their groups one can prevent the failure propagation in the system. The group protections aimed at preventing or mitigating outer impacts cannot usually prevent propagation of failures originated within the group of protected elements to other elements of this group.

In this paper we consider the problem of evaluating the reliability and performance distribution of series–parallel multi-state systems (MSS) with statistically independent selective PFs and imperfect protections with given failure probabilities. The suggested algorithm is based on the universal generating function (UGF) technique [22] and the generalized reliability block diagram (RBD) approach [18].

In this work, the following assumptions are made. The series–parallel system consists of statistically independent multi-state elements, some of which can cause PF. For each PF a set of elements

Abbreviations: CCF, common cause failure; PF, propagated failure; PG, protection group; MSS, multi-state system; RBD, reliability block diagram; UGF, universal generating function (u-function); Pmf, probability mass function

* Corresponding author at: The Israel Electric Corporation, P.O. Box 10, 31000 Haifa, Israel.
E-mail address: levitin@iec.co.il (G. Levitin).

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Notation

n	number of system elements
G_j	random performance of system element j
$\mathbf{g}_j$	set of possible realizations of G_j
g_{jh}	hth realization of G_j
p_{jh}	$\Pr\{G_j = g_{jh}\}$
V	random system performance
v_i	ith realization of V
q_i	$\Pr\{V = v_i\}$
ϕ	system structure function: $V = \phi(G_1, \ldots, G_n)$
θ	system demand
$\pi(V, \theta)$	acceptability function
$R(\theta)$	system reliability: $\Pr\{\pi(V, \theta) = 1\}$
$W(\theta)$	conditional expected system performance
$u_j(z)$	u-function representing unconditional pmf of G_j
$\tilde{u}_j(z)$	u-function representing conditional pmf of G_j given element j does not fail due to PF
$\hat{u}_j(z)$	u-function representing conditional pmf of G_j of individually protected element given it is affected by a PF
$U(z)$	u-function representing pmf of V
$f(z)$	u-function representing conditional pmf of G_j given element j fails
A_h	subset of system elements that can be affected by a PF originated from element h
s	number of combination of PFs
Q_s	probability of combination s of the PFs
X_s	set of elements that cause PFs in combination s
Φ_s	set of elements affected by the combination s of the PFs
Δ_s	set of elements that fail as a result of PFs originated within their PGs given the combination s of PFs occurs
B	number of protection groups in the system
Y_b	set of elements belonging to protection group b
d_b	protection failure probability of PG b
ω	set of unprotected elements (not belonging to any PG)
e	number of combination of protection failures
Ξ_e	set of failed protections corresponding to combination e
Ω_e	set of elements that lose their protection when combination e of the protection failures happens
φ_{se}	probability of combination e of the protection failures given combination s of PFs
$\underset{\varphi}{\otimes}$	composition operator over u-functions
$\varphi(G_i, G_j)$	function representing performance of pair of elements

that can be affected by the PF is determined. Individual elements and groups of elements can be protected. The group of elements with common protection is named a protection group (PG). No system element can belong to more than one PG. If a PF that can affect certain element happens, the element fails only if the protection of the PG it belongs to also fails. The protection cannot fail if its PG is not affected by any PF. If protection does not fail, the element cannot fail because of any PF originated outside its PG. No protection can prevent propagation of failure originated within PG it protects. If an element is not protected initially (does not belong to any PG) it always fails if it is affected by a PF. The PFs are independent events. The failures of protections are independent events for any given combination of PFs.

The remainder of the paper is organized as follows. Section 2 presents the generic model of MSS and describes the UGF technique and generalized RBD algorithm for evaluating the system performance distribution and reliability indices. Section 3 suggests the method to incorporate the PFs and protection failures into the RBD algorithm. The cases of individual and group protections are considered. Section 4 presents analytical and numerical examples of analyzing the performance of multi-state systems. Section 5 concludes.

2. RBD method for MSS

2.1. Generic model of MSS

In order to analyze MSS behavior, characteristics of its elements must be identified. Any system element j can have k_j+1 different states corresponding to the performance rates, represented by the set $\mathbf{g}_j = \{g_{j0}, g_{1j}, \ldots, g_{jk_j}\}$, where g_{jh} is the performance rate of element j in the state h, $h \in \{0, 1, \ldots, k_j\}$. The performance rate G_j of element j at any time instant is a discrete random variable that takes its values from $\mathbf{g}_j$: $G_j \in \mathbf{g}_j$. The probabilities associated with the different states (performance rates) of the system element j can be represented by the set

$$\mathbf{p}_j = \{p_{j0}, p_{j1}, \ldots, p_{jk_j}\}, \tag{1}$$

where

$$p_{jh} = \Pr\{G_j = g_{jh}\}. \tag{2}$$

Since the element's states compose the complete group of mutually exclusive events, meaning that the element can always be in one and only one of k_j+1 states, we have

$$\sum_{h=0}^{k_j} p_{jh} = 1. \tag{3}$$

Eq. (2) defines the probability mass function (pmf) of the random variable G_j. The collection of pairs (g_{jh}, p_{jh}), $h = 0, 1, \ldots, k_j$, completely determines the performance distribution of element j.

The performance rate of an MSS consisting of n independent elements is unambiguously determined by the performance rates of these elements. At each moment, the system elements have certain performance rates corresponding to their states. The state of the entire system is determined by the states of its elements. Assume that the entire system has $K+1$ different states and that v_i is the entire system performance rate in state $i \in \{0, \ldots, K\}$. The MSS performance rate is a random variable V that takes values from the set $M = \{v_0, \ldots, v_K\}$. Define $L^n = \{g_{10}, \ldots, g_{1k_1}\} \times \{g_{20}, \ldots, g_{2k_2}\} \times \cdots \times \{g_{n0}, \ldots, g_{nk_n}\}$. It is actually the space of all possible combinations of performance rates for all of the system elements. The function $\phi(G_1, \ldots, G_n)$: $L^n \to M$ which maps the space of the elements' performance rates into the space of system's performance rates is named the system structure function.

The generic model of an MSS includes the pmf of performances for all of the system elements and the system structure function:

$$\mathbf{g}_j, \mathbf{p}_j, \ 1 \le j \le n, \tag{4}$$

$$V = \phi(G_1, \ldots, G_n). \tag{5}$$

From this model the pmf of the entire system performance can be obtained in the form

$$q_i, v_i, \ 0 \le i \le K, \text{ where } q_i = \Pr\{V = v_i\}. \tag{6}$$