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Likelihood ratio gradient estimation for dynamic reliability applications

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ABSTRACT

This paper investigates the issue of performing a first-order sensitivity analysis in the setting of dynamic reliability. The likelihood ratio (LR) derivative/gradient estimation method is chosen to fulfill the mission. Its formulation and implementation in the system-based Monte Carlo approach that is commonly used in dynamic reliability applications is first given. To speed up the simulation, we then apply the LR method within the framework of Z-VISA, a biasing (or importance sampling) method we have developed recently. A widely discussed dynamic reliability example (a holdup tank) is studied to test the effectiveness and behaviors of the LR method when applied to dynamic reliability problems and also the effectiveness of the Z-VISA biasing technique for reducing the variance of LR derivative estimators.

1. Introduction

"Dynamic reliability" [1,2] has been gaining significant international momentum in recent years. A host of methodologies have been proposed [3,4], including Discrete Dynamic Event Tree (DDET) [5–8], Monte Carlo (MC) simulation [9–12], Cell-to-Cell Mapping Technique (CCMT) [13], etc. These dynamic reliability methods capture explicitly the influence of time and process dynamics on scenarios, which is often difficult in the classical Event Tree/Fault Tree (ET/FT) approach. The modeling and thus the risk assessment results that these dynamic reliability methods produce are believed to be of higher fidelity with respect to reality. However, most research efforts in the field so far have been dedicated to evaluating relevant probabilities, while there has been little work on importance and sensitivity analysis.

In risk and reliability applications, importance and sensitivity analysis is of paramount value as it provides useful information regarding the importance of various components/parameters of a system in view of their risk or safety significance. Such importance and sensitivity information can play an important role in a number of engineering activities/processes. Firstly, it can help risk analysts and managers to identify system vulnerabilities or weak points, and prioritize system parts that need improvement. Proper decisions can be made, based on this knowledge, about investment of resources (time, money, etc.) into relevant activities (design, diagnostics, maintenance, etc.). Secondly, as far as data uncertainty is concerned, such importance and sensitivity information can also provide guidelines for effort allocation in reliability data collection. For example, more efforts are worthy when collecting data of those parts that have larger sensitivities, since large uncertainties in those parts' input data are likely to lead to large uncertainties in the final risk assessment results. For derivative sensitivities, a third class of applications is optimization. One wishes to set various parameters that are controllable to optimize some performance measure. Sensitivity analysis is often the first step in an optimization. For convex optimization and more general optimization algorithms that employ gradient-based search, derivatives are key elements. Another possible application of derivatives is interpolation (see [14] and references therein for more details).

Much work has been done on importance and sensitivity analysis in classical risk and reliability analysis (as opposed to dynamic reliability). A number of importance measures have been developed, such as Birnbaum's Measure, Risk Achievement Worth (RAW), Risk Reduction Worth (RRW), Fussell–Vesely's Measure, and Criticality Importance [15,16]. These importance measures are widely used and play an important role in practice. However, there are shortcomings and limitations to the current importance analysis [16,17]. One such limitation is that it is typically performed at the component level. It is often desirable, however, to investigate the issue of importance at a more elementary level (parameter level, i.e., to evaluate the sensitivities (derivatives) of some system risk/reliability metric with respect to various parameters). One reason is

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that parameter-level sensitivities are more general and have a wider area of applications. Secondly, in practice many engineering activities and decision-makings are carried out at the level of parameters (e.g., failure rates, repair rates). Furthermore, most of the importance measures that were developed for classical reliability analysis are not applicable for the setting of dynamic reliability. At the current time, no mature importance measure exists for dynamic reliability applications, while in principle parameter-level sensitivities are also applicable for dynamic reliability as well as classical reliability analysis.

In this work, we investigate how to evaluate derivatives of a given system risk/reliability metric with respect to relevant parameters in the setting of dynamic reliability. In usual, system risk/reliability metrics (more generally, system performance measures) can be expressed as an expectation of some function of the parameter vector θ and the sample path ω, say $h(\theta,\omega)$, with respect to a probability measure P_θ over some measurable space $(\Omega,\mathcal{F})$

$$r(\theta)=E_{P_\theta}[h(\theta,\omega)]=\int_\Omega h(\theta,\omega)dP_\theta(\omega)=\int_\Omega h(\theta,\omega)f(\theta,\omega)d\omega \quad (1)$$

where E_{P_θ} denotes expectation taken with respect to P_θ, and it is assumed that there exists a probability density function $f(\theta,\omega)$ for the probability measure P_θ. Due to the problem complexity, evaluations of $r(\theta)$ and its derivatives are often analytically and numerically intractable, and one needs to resort to simulation. We focus here on an important class of sample performance functions, $h(\theta,\omega)=I_{\{\omega\in\Re\}}$, namely an indicator function of the event $\{\omega\in\Re\}$, where $\Re$ is some domain of interest in the sample space (e.g., failure/accident domain). It then follows that the risk/reliability metric $r(\theta)$ represents the probability that the system enters some failure/accident domain. Our aim is to estimate, via simulation, the failure/accident probability and its sensitivities (derivatives) to relevant parameters

$$r(\theta)=\int_\Omega I_{\{\omega\in\Re\}}f(\theta,\omega)d\omega \quad (2)$$

$$\frac{\partial}{\partial\theta_i}r(\theta)=\frac{\partial}{\partial\theta_i}\int_\Omega I_{\{\omega\in\Re\}}f(\theta,\omega)d\omega \quad (3)$$

where θ_i is the ith component of the parameter vector $\theta=(\theta_1,\ldots,\theta_d)$, with d being the number of parameters of interest. For notational simplicity, we denote $\partial r(\theta)/\partial\theta_i$ by r_{θ_i} sometimes.

In practice, safety-critical systems such as nuclear power plants, airplanes, spacecrafts, and chemical plants, are usually highly reliable. The probability of the event $\{\omega\in\Re\}$, namely the system enters the failure/accident domain $\Re$, is generally very low. Hence, we are often facing the well-known rare event simulation challenge. Without proper variance reduction techniques, an excessive number of histories need to be simulated before achieving a satisfactory statistical accuracy, thus resulting in an unacceptably long computing time. This problem may become even more formidable for dynamic reliability applications which involve physical processes and might need a significant computer time for simulating even a single history due to the heavy computation burden incurred by the calculation of system dynamics. Therefore, we need to take into account the computational cost challenge (mainly the variance issue) when estimating system risk/reliability metrics and their derivatives.

The rest of this paper is organized as follows. Section 2 gives a brief description of dynamic reliability. In Section 3, we investigate the main derivative/gradient estimation techniques that exist in the literature and conclude that the likelihood ratio (LR) method fits our context well. Applications of the LR method within the commonly adopted system-based analog Monte Carlo approach and within the framework of Z-VISA are presented in Sections 4 and 5, respectively. A well-known dynamic reliability example (a holdup tank) is studied in Section 6. The final section offers some concluding remarks.

2. Dynamic reliability

In a dynamic reliability context, the evolution of the system is significantly affected by an underlying process dynamics (process variables such as temperature, pressure, and liquid level). The two constituents of the system, hardware components (also software and human elements, more generally) and process variables, together determine the evolution of the system through interacting with each other. On the one hand, modification of the system's hardware configuration (due to stochastic failures of hardware components or actions of control/protection devices) may change the mode of process variables' evolution; on the other hand, the evolution of process variables can induce operations of control/protection devices upon reaching preset thresholds; process dynamics may also affect hardware components' stochastic characteristics (e.g., a high temperature leads to increased component failure rates). Therefore, an integrated treatment needs to be made for hardware components, process dynamics, and their interactions when dealing with such dynamic reliability problems. Another characteristic of dynamic reliability induced by the involvement of process dynamics is that the definition of system failure is not restricted to the system transitioning to some failed hardware configurations as in classical reliability analysis. It could also be a crossing of the boundary of a safety domain in the space of process variables.

As the system evolves with time, two different types of transitions can occur: transitions in operation and transitions on demand.[1] The first category of transitions is characterized by: the timing of transitions is random and can usually be described by a continuous probability distribution. Examples of such transitions are failure of a component, repair of a component, etc. The second category of transitions differs from the first one in that: the timing of transitions is not random while the outcome could be either deterministic or random, depending on different assumptions. If the outcome is considered to be random, it can often be described by a discrete probability distribution. An example of such a transition is the switching of a valve demanded by the control/protection system when the pressure or liquid level reaches a preset threshold. If one assumes that the valve will always switch successfully, then the outcome is deterministic; but if a non-zero failure probability of switching is assumed, the outcome would then be random.

The above two different types of transitions modify the system's hardware configuration, which may lead to changing the evolution mode of the underlying process dynamics. Thus, as the system evolves, it switches randomly from one dynamics to another. (Hence, the term "probabilistic dynamics" is sometimes used as an alternative for "dynamic reliability".) More formally, let the hardware configuration of the system, which can be identified by the states of all components, be indexed by an integer i. Let the vector of process variables be denoted by $\mathbf{x}$ ($\mathbf{x}\in\mathbb{R}^d$, with d being the number of process variables). The state of the whole system can be identified by the pair $(i,\mathbf{x})$. In general, the evolution of process variables in a certain configuration follows a physical model depending on the specific configuration,

[1] Some other terms that have similar meanings are also used in the literature, e.g., transitions during operation, transitions in time, time-based transitions for the first type of transitions, and transitions upon demand, demand-based transitions for the second type of transitions.