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Geometric and kinematic analysis of a seven-bar three-fixed-pivoted compound-joint mechanism

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ABSTRACT

This paper investigates a seven-bar three-fixed-pivoted compound-joint mechanism and presents a systematic classification based on the rotatability criterion of its various types evolved from the change of link parameters. By decomposing the mechanism into two closed loops that provide respectively position and orientation of the end-effector link, closed form kinematic equations are developed using two four-bar analytical equations in sequence for geometric analysis of the mechanism. The paper further derives the Jacobian matrices of the mechanism and presents its kinematic analysis. In the extended group of this mechanism, the paper investigates four typical mechanisms, examines their workspaces and singularity based on the Jacobian matrices and implements the condition number analysis to identify the isotropy distribution.

1. Introduction

Interest in using closed-loop linkages as robotic manipulators has arisen in the past two decades. Closed-loop manipulators offer advantages of eliminating gear trains and facilitating other drive transmissions that are usually demanded in open-loop manipulators. Applying closed-loop planar linkages, particularly the two-dof closed-loop planar linkages to robotic field began from late 1980s. In 1985, Asada and Ro [1] introduced the direct drive to a two-dof five-bar compound-joint planar mechanism to overcome the problem faced by the open-loop mechanism. In 1986, Bajpai and Roth [2] analyzed the influence of the link lengths on the reachable workspace of the same two-dof five-bar manipulator whose classification and effect of orientation of the wrist on the floating link were further presented by Ting [3] in 1992.

Since 1990s, more two-dof five-bar planar linkages were proposed and their geometry and kinematics were examined [4–8]. Further to these mechanisms, the seven-bar mechanism as a type of two-dof planar mechanisms started drawing attention from the mechanism and robotics community. In 1996, Gosselin [9] developed a seven-bar linkage by adding a crank to the Watt II six-bar linkage to form a three-legged planar parallel mechanism with one ternary link as its platform and analyzed its kinematics and static characteristics. Using Assur group, Innocenti [10,11] presented the position analysis [12,13] for synthesis of a three-looped Assur seven-bar mechanism with respect to its assembly configurations. Type synthesis of a two-dof seven-bar planar mechanism with fixed-pivoted single joints was carried out by Fang and Zou [14] that reveals four types of linkages. Further to this, Balli and Chand [15] proposed a synthesis method of a three-pivoted planar seven-bar mechanism using dead-center positions with variable topology analysis to reduce the solution space. In 2003, a two-dof

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Nomenclature

l_1 through to l_7	length of the respective links of the seven-bar mechanism
$l_{3'}$	length of DG link
$l_{7'}$	length of the extension bar of link 7
$\boldsymbol{p}_A, \boldsymbol{p}_B, \ldots, \boldsymbol{p}_H$	coordinate vectors of the joints in the seven-bar mechanism
$\theta_1, \theta_3, \theta_5$, and θ_7	angular displacements of corresponding links in the seven-bar mechanism
$\dot{\theta}_1, \dot{\theta}_3, \dot{\theta}_5$, and $\dot{\theta}_7$	angular velocities of the corresponding links in the seven-bar mechanism
x_H, y_H	Cartesian coordinates of the wrist point H in the seven-bar mechanism
$\dot{x}_H, \dot{y}_H$	velocity components of the wrist point H in the seven-bar mechanism
$\mathbf{J}$	Jacobian matrix of the seven-bar mechanism
$\mathbf{J}_q, \mathbf{J}_\theta$	the forward and inverse Jacobian matrices of the seven-bar mechanism
$\kappa(\mathbf{J})$	condition number of the seven-bar mechanism

seven-bar mechanism with six revolute pairs and one prismatic pair was developed by Du and Guo [16] for a new metal forming press. In 2005, Luo and Dai [17] put forward a new kind of seven-bar mechanisms with a parallelogram loop and examined their geometry. In 2006, Tong [18] used the concept of orthogonal paths to investigate a three-looped seven-bar mechanism which was developed from the Stephenson III six-bar linkage.

In the process of mechanism analysis, a classification helps identify the common characteristics. The concept of mechanism classification was first put forward by Willis [19]. While, for planar linkages, the most useful method of classification is to use Grashof's criteria [20]. A typical way of a classification was shown by Bajpai and Roth [2] in the five-bar compounded joint mechanisms that were specified as two classes according to their rotatability [21,22] of the driving links. Each of the classes was further categorized into two types. The rotatability study was extended by Dai and Paresh [23] to kinematics chains by introducing a virtual link and the kinematic chains were then classified according to their rotatability. Another typical case of a classification planar 4R linkages was presented by McCarthy [24] with eight types according to the signs of the parameters T_1, T_2, T_3 that are derived from the quadric equation of a four-bar linkage to determine the Grashof and non-Grashof linkages. The combination of their signs is used to determine the crank-rocker condition in the classification.

It has been proposed by an enumeration process that there exists only three two-DOF seven-bar kinematic chains [25–27]. The above study, except for [17], are all based on fixed-pivoted single-joint linkages with only two links at each of fixed pivoted joints. Most of these mechanisms are either the mechanisms that formed by adding an additional link to the Watt II and Stephenson III six-bar linkages [28,29] or that formed by adding a two-link Assur group to the basic five-link two-dof linkage in all non-isomorphic ways.

This paper presents a seven-bar three-fixed-pivoted compound-joint mechanism which is generated from a combination of two loops one of which utilizes the link contraction operation over a two-dof mechanism while preserving the degree of freedom property. With the link contraction, two of its fixed pivots are located at the same point forming a compound joint. Thus the second loop presents a five-bar linkage with a diminished ground link as part of the seven-bar mechanism. The other part of the seven-bar mechanism is formed by a crank and a coupler which together with the frame form a four-bar linkage and are used to change the position of the end-effector link of the mechanism. Two inputs of the seven-bar mechanism are therefore decoupled to provide one for position and another for orientation. The mechanism gives noncircular trajectories and could be used in origami folding [30] and may further be used for robotic fingers [17] and manipulators. This paper investigates and analyses the mechanism by decomposing it into two closed loops, leading to the classification of its 32 various types. Based on the classification, the paper picks up four typical types that may potentially be used in the robotics field and explores their workspaces, singularity and isotropy distribution.

2. Classification of the mechanisms

The 2-dof seven-bar three-fixed-pivoted compound-joint mechanism has the structure as indicated in Fig. 1.

One pivot of the mechanism is fixed at joint A, the other two fixed pivots are at fixed joint D which form a compound joint. The lengths of links 1 through to 7 are denoted by $l_1, l_2, l_3, l_4, l_5, l_6, l_7$, the length of DG is denoted as $l_{3'}$ and the extension of link 7 as $l_{7'}$. This seven-bar mechanism can be decomposed into two closed loops. The first loop ABCDA (links 1–4) acts as a drive loop and the second loop DEFGD (links 3 and 5–7) acts as an orientation loop that controls the orientation of the coupler link or end-effector link 7 with its extension $l_{7'}$. The second loop is a five-bar linkage with understanding that there is a diminished link at the compound joint D. It should be noted that link 3 is shared by both two loops as a shared link and its posture is determined by the first loop. Once it is fixed in the first loop, the second loop becomes a four-bar linkage. In this structure, the workspace of this mechanism is determined by coupler/end-effector link 7. The position of the coupler link 7 is determined by the first loop while using the first loop crank link 1 and the first loop coupler link 2. These links form the four-bar linkage that substantiates the main five-bar linkage of the second loop to become a seven-bar mechanism with three-fixed pivots.