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A geometrical alternative to Jacobian rank deficiency method for planar workspace characterisation

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ABSTRACT

The workspace of a serial manipulators is defined as the area covered by its extremity for a finite number of degrees of freedom. In order to define its boundaries, a pure geometrical interpretation of the jacobian's property is applied, in opposite to the specific methods or symbolic calculation as proposed in the literature. This solution leads to a general and exact description of the boundaries, independent of the number of degrees of freedom in the planar case and compatible with real-time computation as required in robotics.

1. Introduction

The workpace of a serial manipulators is defined as the area covered by its extremity for a finite number of degree of freedom. The boundary of the workspace is called the "reach envelop" [1]. The characterisation of the workspace's boundaries could be applied either in robotics or in biomechanics. The illustration will take place in this last field through the human upper limb extremity (i.e. fingertip) displacement. The workspace's boundaries present a complex shape due to the number of segments and joints' degrees of freedom. This knowledge is necessary in order to simulate and to analyse any human limb displacements, such as in reaching and pointing tasks.

From a mathematical point of view, in dimension $n \in \{2, 3\}$, the workspace is considered as the range of a convex polytope of $\mathbb{R}^p$ $(n \leqslant p)$ by a differentiable function Φ_p. Here p is the number of independent parameters. Classically, this boundary is determined using the Jacobian of function Φ_p which is necessary singular at the boundaries [2–6]. This can be resolved by vanishing all determinants of the Jacobian. Many studies have been conducted on this fundamental problem for series manipulators [7–13]. However, their results did not propose any general resolution. They only concerned specific cases (small value of degrees of freedom) or numerical approximated descriptions. On the other hand, some authors gave a general method, based on symbolic calculation [2–5,14]. This symbolic calculation such used in the current swept volume approach can be used in robotics, as all robot's data are known; first, the calculus are off-line made in symbolic, then numerically

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implemented. Nevertheless, for the future optimization in biomechanics, human parameters being unknown, the numerical methods will be applied. Thus, the aim of this paper was to propose a pure geometrical solution as an alternative of the Jacobian's property on the boundaries. Therefore, this problem has been solved in regard to this geometrical interpretation, allowing the use of a pure analytical method without symbolic calculation.

We do not extend our results to the 3D, in this paper, for three reasons: first, to our knowledge, this method of geometrical characterisation has not been used previously in the literature and it seems important to define this geometrical method in 2D. Secondly, an important aim of this paper is to present a method, which can be applied to biomechanics problem (see Section 5). For this, in a first step, only the workspace in 2D is required. Third, the extension of this method to the 3D would be considered, with the same ideas as in 2D. Nevertheless, on a technical point of view, the presentation is longer and will be presented in a future paper.

The theoretical basis and the study problem will be presented in Section 2. Then, in Section 3, the main results of this paper will be provided. These results will be used in Section 4, in order to define finite union of arc of circles which contains the boundaries. Finally, Section 5 will include some numerical simulations. Usual recalls about theoretical and technical details will be given in Appendices.

2. Theoretical basis and presentation of the studied problem

Let $(O,\vec{i},\vec{j})$ be a reference frame (which is a direct orthonormal basis), p an integer greater than or equal to 2, $(l_i)_{1\leqslant i\leqslant p}$, p non negative numbers and $(\theta_i^+)_{1\leqslant i\leqslant p}$ and $(\theta_i^-)_{1\leqslant i\leqslant p}$ $2p$ angles satisfying

$$\forall i \in \{1,\ldots,p\}, \quad -\pi < \theta_i^- < \theta_i^+ \leqslant \pi. \tag{1}$$

For all figures, θ_i's are chosen counter-clockwise. They are algebric angles; thus, representation could be either counter-clockwise or clockwise.

We define the workspace as the set of points A_p such as (see Fig. 1)

$$A_0 = 0, \tag{2a}$$

$$\left(\widehat{\vec{j},\overrightarrow{0A_1}}\right) = \theta_1, \tag{2b}$$

$$\forall i \in \{2,\ldots,p\}, \quad \left(\widehat{\overrightarrow{A_{i-2}A_{i-1}},\overrightarrow{A_{i-1}A_i}}\right) = \theta_i, \tag{2c}$$

$$\forall i \in \{1,\ldots,p\}, \quad A_{i-1}A_i = l_i, \tag{2d}$$

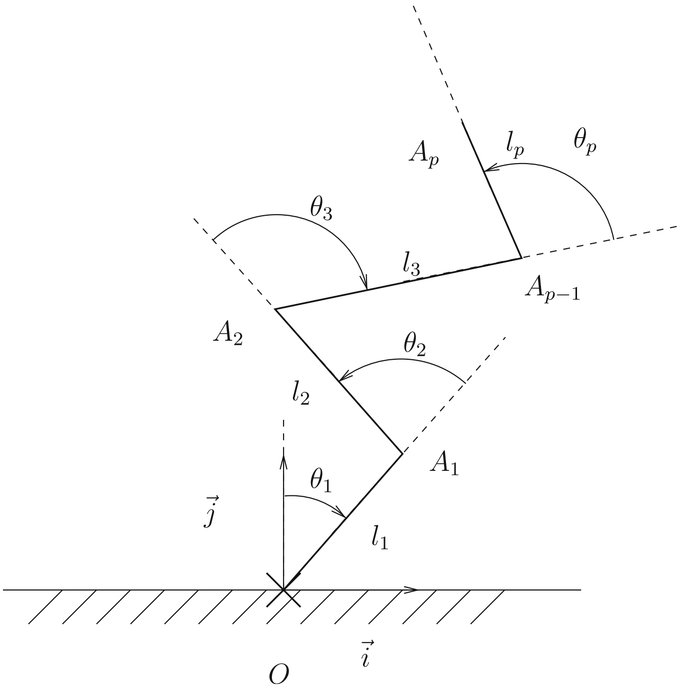

Fig. 1. The considered planar system.