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An extension of mechanism design optimization for motion generation

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ABSTRACT

As an extension of the authors published work on a search algorithm for motion generation with Grashof, transmission angle and linkage perimeter conditions [P.J. Martin, K. Russell, R.S. Sodhi, On mechanism design optimization for motion generation, Mechanism and Machine Theory 42 (10) (2007) 1251–1263], this work formulates a goal program to generate four-bar mechanism fixed and moving pivot loci that considers prescribed coupler poses, a coupler load and maximum driver static torques.

1. Introduction

In planar four-bar **motion generation**, the objective is to calculate the mechanism parameters required to achieve or approximate a set of prescribed coupler poses. This mechanism design objective is particularly useful when the coupler must achieve a specific displacement sequence for effective operation (e.g., specific end-effector orientations for accurate task completion). In Fig. 1, five prescribed coupler poses are defined by the x and y-coordinates of variables $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ (Fig. 1a) and the calculated mechanism parameters to achieve the prescribed poses are the x and y-coordinates of fixed pivot variables $\mathbf{a}_0$ and $\mathbf{b}_0$ and moving pivot variables $\mathbf{a}_1$ and $\mathbf{b}_1$ (Fig. 1b). The latter figure illustrates a planar four-bar motion generator.

Motion generation for planar four-bar mechanisms is a well-established field. Recent contributions include the work of Yao and Angeles [3] who applied the contour method in the approximate synthesis of planar linkages for rigid-body guidance. By deriving a set of two bivariate polynomial equations and plotting these equations, the real solution to the optimizations-which correspond to the intersections of the contour plots are determined. Hong and Erdman [4] introduced a new application Burmester curves for adjustable planar four-bar linkages. Their method is applicable for the synthesis of four-bar mechanisms is planar and spherical form and their work shows that nonadjustable mechanism solutions are special cases of adjustable mechanism solutions. Zhou and Cheung [5] introduced an optimal synthesis method of adjustable four-bar linkages for multi-phase motion generation. A modified genetic algorithm is used to seek the global optimal solution of an equation set that includes constraints for fixed pivot positions, no branch defect, crank existence and link length ratios. Al-Widyan et al. [6] considered the robust synthesis of planar four-bar linkages for motion generation. Danieli et al. [7] applied Burmester theory in the design of planar four-bar motion generators to reproduce tibia-femur relative motion. Goehler et al.

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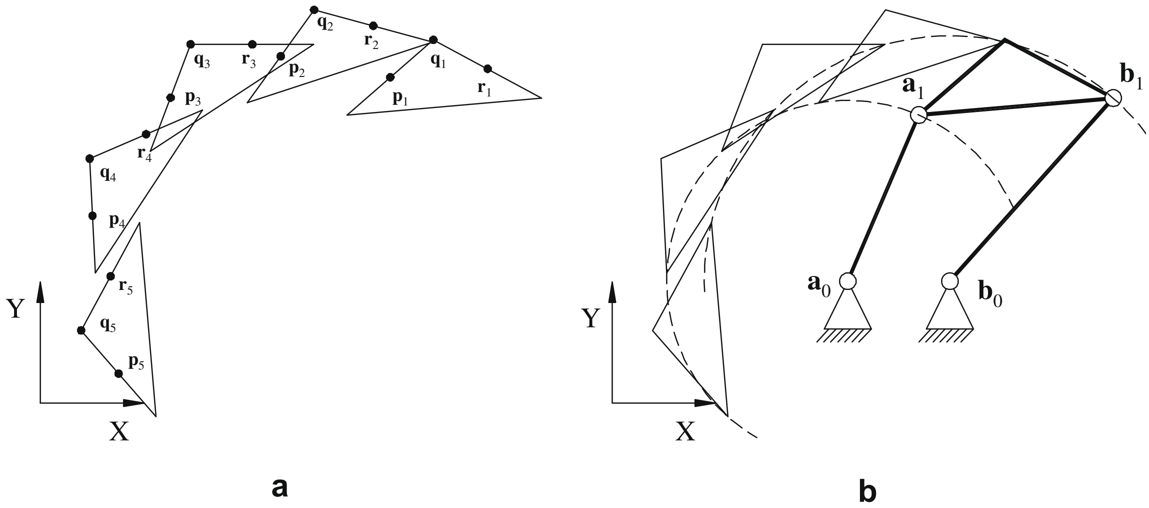

Fig. 1. Prescribed coupler poses (a) and calculated planar four-bar mechanism (b).

[8] applied parameterized T_1 motion theory to the synthesis of planar four-bar motion generators. This T_1 motion theory is general and not limited to the second order parameterization that is associated with prior development of T_1 motion theory. Caracciolo and Trevisani [9] considered rigid-body motion control of flexible four-bar linkages. A discrete finite element model of the four-bar mechanism accounts for geometric and inertial nonlinearities. Zhixing et al. [10] presented a guidance-line rotation method of rigid-body guidance for the synthesis of planar four-bar mechanisms. The method effectively solves the rigid-body guidance synthesis problem crank–rocker mechanisms, double-rocker mechanisms and double-crank mechanisms for four, five or more than five rigid-body positions. Lin and Modler [11] presented a method to avoid branch defects, order defects and ensure link rotatability in three-point path generation. The method considers (but is not limited to) planar four-bar mechanisms.

Using a conventional motion generation model [12] the user can only calculate the planar four-bar mechanism parameters required to achieve or approximate a limited set of prescribed coupler poses. Although such solutions are effective for kinematic analyses, other design factors (e.g., static loads, deflections, stresses, strains, etc.) must be considered for a comprehensive engineering analysis. This work considers static driving link torque for a given coupler load for motion generation. Unlike previously published work on analytical motion generation with prescribed coupler loads, where the number of prescribed coupler poses is limited and the mechanism solution is exact [2], the numerical model formulated in this work can accommodate an indefinite number of prescribed coupler poses that are approximated by the mechanism solution. By incorporating the calculated coupler load and driver static torque-based fixed and moving pivot loci in the search algorithm, planar four-bar motion generator solutions are sought that also satisfy specified Grashof conditions, transmission angle conditions and mechanism perimeter conditions.

2. Conventional planar four-bar motion generation

Eqs. (1) and (2) encompass the numerical planar four-bar motion generation model presented by Suh and Radcliffe [12]. Eqs. (1) and (2) ensure the constant lengths of the crank and follower links. Eq. (3) is a planar rigid-body displacement matrix.

When using this conventional mechanism synthesis model to calculate the components of the fixed pivots $\mathbf{a}_0$ and $\mathbf{b}_0$ (where $\mathbf{a}_0 = [a_{0x}, a_{0y}, 1]^{\mathrm{T}}$, and $\mathbf{b}_0 = [b_{0x}, b_{0y}, 1]^{\mathrm{T}}$) and moving pivots $\mathbf{a}_1$ and $\mathbf{b}_1$ (where $\mathbf{a}_1 = [a_{1x}, a_{1y}, 1]^{\mathrm{T}}$, and $\mathbf{b}_1 = [b_{1x}, b_{1y}, 1]^{\mathrm{T}}$) of a planar four-bar motion generator (Fig. 1b), the user can specify a maximum of five coupler poses [12]:

$$\left([\mathbf{D}_{1j}]\mathbf{a}_1 - \mathbf{a}_0\right)^T\left([\mathbf{D}_{1j}]\mathbf{a}_1 - \mathbf{a}_0\right) - (\mathbf{a}_1 - \mathbf{a}_0)^T(\mathbf{a}_1 - \mathbf{a}_0) = 0, \tag{1}$$

$$([\mathbf{D}_{1j}]\mathbf{b}_1 - \mathbf{b}_0)^T([\mathbf{D}_{1j}]\mathbf{b}_1 - \mathbf{b}_0) - (\mathbf{b}_1 - \mathbf{b}_0)^T(\mathbf{b}_1 - \mathbf{b}_0) = 0, \tag{2}$$

where

$$[\mathbf{D}_{1j}] = \begin{bmatrix} p_{jx} & q_{jx} & r_{jx} \\ p_{jy} & q_{jy} & r_{jy} \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} p_{1x} & q_{1x} & r_{1x} \\ p_{1y} & q_{1y} & r_{1y} \\ 1 & 1 & 1 \end{bmatrix}^{-1} \quad (j = 2,3,4,5). \tag{3}$$