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Effect of addendum on contact strength, bending strength and basic performance parameters of a pair of spur gears

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Abstract

This paper investigates effect of addendum on tooth contact strength, bending strength and basic performance parameters of spur gears. Face-contact model of teeth, mathematical programming method (MPM) and three-dimensional (3D), finite element method (FEM) are used together to conduct loaded tooth contact analyses (LTCA), deformation and stress calculations of spur gears with different addendums and contact ratios. Tooth load, load-sharing rate, contact stress, root bending stress, transmission error and mesh stiffness of the spur gears are analyzed. Effects of addendum and contact ratio on gear strength and basic performance parameters are also discussed.

It is found that contact stress distributed on tooth surface along the profile is asymmetrical around the geometrical contact point of the gears when the geometrical contact point is far away from the pitch point, especially at the tip or root contacts. The maximum contact stress is also not exactly at the geometrical contact point. Hertz formula can only calculate the tooth contact stress approximately. FEM can be used to conduct precise analysis of the tooth contact stress.

It is also found that increment of addendum can increase number of contact teeth, then this increment can reduce tooth contact stress and root bending stress, generally speaking. But this increment also makes the tooth depth long and lets the tooth be deformed easily. So, this increment can reduce tooth contact stress, but there is no guarantee that this increment can certainly reduce tooth root bending stress. Also, increment of the addendum makes the values of "*T*" and "*V*" in the so-called value of "*PVT*" great. So, scoring strength calculations of the tooth tip and root becomes more important when to use high contact ratio gears (HCRG) with long addendum.

Finally, strength calculations of HCRG with misalignment error, lead crowning and thin-walled thickness are discussed simply in this paper.

Keywords: Gear; Spur gear; Tooth load; Contact stress; Root stress; Mesh stiffness; Transmission error

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Nomenclature

$a_{kj(1)}, a_{kj(2)}$	deformation influence coefficients of the reference points on the contact reference face of the pinion and gear
c_k	tooth clearance coefficient
k	addendum coefficient
k_G	tooth mesh stiffness
m	module of gears
P	total load of the pair of gears along the line of action
r_b	radius of gear base circle
$\{F\}$	array of contact loads between the pairs of assumed contact points
W_H	contact width
δ	the total relative deformation of the pair of gears along the line of action
θ	transmission error of a pair of gears
$\{\varepsilon\}$	gap array that consists of all pairs of contact points
ε_k	gap of the pair of contact reference points $(k-k')$
ε	contact ratio of gears
$(k-k')$	assumed pair of contact points on the reference faces of pinion and gear
$\{Y\}$	slack variables, $\{Y\}=\{Y_1 \quad Y_2 \quad \cdots \quad Y_k \quad \cdots \quad Y_n\}^T$
X_{n+1}	artificial variables, also, X_{n+2}, $X_{n+n}, \ldots, X_{n+n+1}$
$[I]$	unit matrix of $n \times n$, n is size of the unit matrix
Z	objective function
$[S]$	matrix of the deformation influence coefficients
$\{F\}$	array of contact force of the pairs of contact points, $\{F\}=\{F_1 \quad F_2 \quad \cdots \quad F_k \quad \cdots \quad F_n\}^T$
$\{\varepsilon\}$	array of clearance of the pairs of contact points, $\{\varepsilon\}=\{\varepsilon_1 \quad \varepsilon_2 \quad \cdots \quad \varepsilon_k \quad \cdots \quad \varepsilon_n\}^T$
$\{e\}$	unit array, $\{e\}=\{1 \quad 1 \quad \cdots \quad 1 \quad \cdots \quad 1\}^T$
$\{0\}$	zero array, $\{0\}=\{0 \quad 0 \quad \cdots \quad 0 \quad \cdots \quad 0\}^T$
MPM	mathematical programming method
2D	two-dimensional
3D	three-dimensional
FEM	finite element method
FEA	finite element analysis
LTCA	loaded tooth contact analyses
HCRG	high contact ratio gears
HCR	high contact ratio
①	pinion
②	gear

1. Introduction

High contact ratio gears (HCRG) are usually used in airplanes [1] to transmit large torque. But in recent years, these gears have been finding wide applications in cars, industry robots and other machines for noise and vibration reduction. Though it was very early to use the HCRG in machines, strength calculation problems of these gears have not been solved throughout in theory. Gear designers often use ISO standards [2–4] to do strength calculations of these gears approximately.

Many papers [5–9] discussed the effect of high contact ratio (HCR) on dynamic behavior of spur gears, but only a few deal with strength calculation of the HCRG. Eikholy [10] analyzed tooth load-sharing of the HCRG by assuming that the sums of the tooth deflections at each pair of contacts are equal and the sum of the normal loads contributed by each pair of contacts is equal to the maximum normal load firstly, then dividing total deflection of a pair of teeth into four parts: tooth shear deflection; bending deflection; tooth