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Nonlinear buckling of eccentrically stiffened functionally graded toroidal shell segments under torsional load surrounded by elastic foundation in thermal environment

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ABSTRACT

The nonlinear buckling and post-buckling of functionally graded stiffened toroidal shell segment surrounded by elastic foundation in thermal environment and under torsional load are investigated in this paper. The functionally graded toroidal shell segments are reinforced by ring and stringer stiffeners system in which material properties of shell are assumed to be continuously graded in the thickness direction. The two-parameter elastic foundation proposed by Pasternak is studied. Theoretical formulations are derived basing on the classical shell theory with the geometrical nonlinearity in von Karman sense and the smeared stiffeners technique. The three-term approximate solution of deflection is chosen more correctly and the explicit expression for finding critical load and post-buckling torsional load-deflection curves are found. The effects of geometrical parameters, temperature, stiffeners and elastic foundation are investigated.

1. Introduction

The concept of functionally graded materials (FGMs) was reported for the first time by Japanese scientists in 1984 [1]. FGMs are composite materials and microscopically inhomogeneous whose mechanical properties vary smoothly and continuously from one surface to the other. Mechanical and physical features of FGMs are better than fiber reinforced laminated composite materials with advantages of no stress concentration, high toughness, oxidation resistance and heat-resistance. FGMs were initially used in thermal barrier materials for aerospace structures and fusion reactors where high temperature or temperature gradient exists. FG shells are also used for storage tanks, pressure vessels and engine industries. Postbuckling and vibration problems of FGM structures in these practical applications are extremely important and therefore they have been received considerable interest of many authors worldwide.

Nowadays, the torsional problem have been attracted much more attention. Sofiyev and Schnack [2] investigated the stability of functionally graded cylindrical shell subjected to torsional loading varying as a linear function of time. In their work, the modified Donnell type dynamic stability and compatibility equations were applied. In the other publications, Sofiyev et al. [3,4] carried out the torsional vibration and buckling analysis of cylindrical shell surrounded by an elastic medium. The torsional stability analysis for thin cylindrical with the functionally graded middle layer resting on the Winkler elastic foundation was implemented by Sofiyev and Adiguzel [5]. The fundamental relations and basic equation of three-layered cylindrical shells with a FG middle layer resting on the Winkler elastic foundation under torsional load were derived. The torsional postbuckling analysis of FGM cylindrical shells in thermal environment based on a higher order shear deformation theory with a von Karman–Donell type of kinematic non-linearity was done by Shen [6]. Wang

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et al. [7] pointed out the exact solutions and transient behavior for torsional vibration of functionally graded finite hollow cylinders. The torsion of a circular cylindrical bar made of either isotropic compressible or isotropic incompressible linear elastic material whose Young moduli vary only in the axial direction was taken into account by Batra [8]. Tan [9] developed the torsional buckling loads of thin and thick shells of revolution based on the classical thin shell theory and the first-order shear deformation shell theory. The torsional analysis of functionally graded hollow tubes of arbitrary shape based on governing equations in terms of Prandtl's stress function was performed by Arghavan and Hematiyan [10]. Zhang and Fu [11] addressed the torsional buckling characteristic of an elastic cylinder with a hard surface coating layer using the Navier's equation and thin shell model. The nonlinear buckling problem of FGM cylindrical shells under torsion load based on the nonlinear large deflection theory by using the energy method and the nonlinear strain-displacement relations of large deformation are studied by Huang and Han [12]. Recently, Dung and Hoa [13,14] have investigated the nonlinear buckling and post-buckling of functionally graded stiffened thin circular cylindrical shells surrounded by elastic foundation in thermal environments under torsional load using analytical approach based on the classical shell theory with the geometrical nonlinearity in von Karman sense.

The FGM structures which have heat-resistant feature were initially found by Noda [15]. Praveen et al. [16] considered the material properties depending on temperature in thermo-elastic analyses of functionally graded ceramic-metal cylinder significantly. Haddadpour et al. [17] performed free vibration analysis of functionally graded cylindrical shells using the equations of motion based on Love's shell theory and the von Karman-Donnell type of kinematic nonlinearity. In their work, the thermal effects investigated by specifying arbitrary high temperature on the outer surface and ambient temperature on the inner surface of the cylinder were considered. Kadoli and Ganesan [18] carried out the linear thermal buckling and free vibration for functionally graded cylindrical shells subjected to a clamped-clamped boundary condition with temperature-dependent material properties. Moreover, Noda and Shen [19] gave the post-buckling analysis for FGM hybrid cylindrical shells in thermal environments. The heat conductivity and material's temperature-dependent properties are calculated. The post-buckling analysis of FGM cylindrical shells under axial compression and thermal loads using the element-free kp-Ritz method was realized by Liew [20]. The thermal instability of deep FGM spherical shells based on the first-order theory and the Sanders nonlinear kinematics equations were studied by Shahsiah et al. [21]. Three types of thermal loadings including the uniform temperature rise, the linear temperature, and the nonlinear radial temperature were given. Boroujerdy and Eslami [22] investigated the FGM shallow spherical shells under thermomechanical instability and surface-bonded piezoelectric actuators. The post-buckling analysis of imperfect stiffened laminated cylindrical shell under combined external pressure and thermal loading using the formulation based on a boundary layer theory of shell buckling including the effects of nonlinear prebuckling deformations, nonlinear large deflection in the postbuckling range and initial geometrical imperfections of the shell was carried out by Shen [23]. Shariyat [24] studied the dynamic buckling of imperfect FGM cylindrical shells with integrated surface-bonded sensor and actuator layers subjected to some complex combinations of thermo-electro-mechanical loads. The general form of Green's strain tensor in curvilinear coordinates and a high-order shell theory proposed earlier by Shariyat was used. Bich et al. [25–27] implemented the nonlinear analyses of functionally graded shallow spherical shells including temperature effects.

The structures on the elastic medium have been researched and developed for a long time by many scientists. The simplest model for the elastic foundation is Winkler-model [28] which is represented by a series of separated springs without any coupling effects between them. This model was then enhanced by integrating a shear layer to it proposed by Pasternak [29]. Duc et al. [30] discussed an analytical approach for the nonlinear static buckling and postbuckling of imperfect eccentrically stiffened FGM of shell structures resting on elastic foundations. The vibration analysis of FGM truncated and complete conical shell on elastic foundation under various boundary conditions was given by Sofiyev and Kuruoglu [31]. The displacement and Airy stress function were sought depending on a new parameter. By applying the Galerkin method to the foregoing equations, the dimensionless frequency parameters of FGM conical shells on Pasternak foundation for two boundary conditions were obtained. Bagherizadeh et al. [32] investigated the mechanical buckling of functionally graded material cylindrical shells surrounded by Pasternak elastic foundation based on a higher-order shear deformation shell theory. Sofiyev [33,34] and Shen [35,36] studied the buckling of FGM shells resting on an elastic foundation.

As can be seen, toroidal shells have been used in such applications as satellite support structures, fusion reactor vessels, rocket fuel tanks, diver's oxygen tanks and underwater toroidal pressure hull. Nowadays FGM consisting of metal and ceramic constituents have received remarkable attention in structural applications. The smooth and continuous change in material properties enable FGM to avoid interface problems and unexpected thermal stress concentrations. Some components of the above-mentioned structures components may be made of FGM. Stein and McElman [37] pointed out the homogenous and isotropic toroidal shell segments with the buckling problem. McElman [38] carried out the eccentrically stiffened shallow shells of double curvature with the static and dynamic behaviors in NASA technical note. The initial post-buckling behavior of toroidal shell segments subject to several loading conditions based on the basic of Koiter's general theory was performed by Hutchinson [39]. Parnell [40] proposed a simple technique for the analysis of shells of revolution applied to toroidal shell segments. Recently, there have had some new publications about toroidal shell segment structure. Bich et al. [41] has studied the buckling of eccentrically stiffened functionally graded toroidal shell segment under axial compression, lateral pressure and hydrostatic pressure based on the classical thin shell theory, the smeared stiffeners technique and the adjacent equilibrium criterion. Furthermore, the nonlinear buckling and post-buckling problems of ES-FGM surrounded by an elastic medium under torsional load based on the analytical approach are investigated by Bich et al. [42].

To the best of authors' knowledge of this paper, there has not been any study on the nonlinear torsional buckling of eccentrically stiffened FGM toroidal shell segments in thermal environment.

In the present paper, the nonlinear torsional buckling and post-buckling of eccentrically stiffened FGM toroidal shell segments, resting on an elastic foundation and considering temperature effects are investigated. The governing equations are derived based on the classical shell theory with the nonlinear strain-displacement relation of large deflection. Moreover, the three-term solution of deflection including the linear buckling and nonlinear buckling shape are more correctly chosen. The Galerkin method is used for nonlinear buckling analysis of shells to give closed-form expressions of the critical buckling load and the relation between deflection and torsional load. Effects of buckling modes, geometrical parameters, volume fraction index, elastic foundation and temperature on the nonlinear torsional buckling behavior of shells are investigated.