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Identification of imperfections in thin plates based on the modified potential energy principle

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ABSTRACT

A procedure to identify the imperfection in thin plates is proposed in this paper. The modified potential energy principle, which serves as the theoretical basis of the identification procedure, is improved to allow for the experimental measurements in static tests. Several typical examples are studied to illustrate the effectiveness of the procedure.

1. Introduction

Imperfection identification is often of foremost concern for many structural systems in service. Non-destructive load tests are usually conducted to determine the unknown parameters for identification of the imperfections.

As a major class of techniques that has the merit of uninterrupted operation of systems, dynamic damage identification methods have been developed for many years (for example, [1,10,13,15,16,26,20]). Among them, some are exclusively proposed to detect the imperfections in thin plates (see [11,18,19]). It has been recognized that the major deficiency of dynamic identification methods is the presence of uncertainties in masses and damping. In contrast, static identification procedures can bypass this deficiency and enjoy easy implementation for simple structural systems. Existing methods based on the static measurements are mostly represented as constrained non-convex optimization problems [2,3,5,12,17,21]. Nevertheless, these static identification procedures have the disadvantage of lack of test repeatability or generality for various structural systems. In addition, neither existing dynamic nor static identification procedures give analytical expressions of identification parameters.

In the recent years, Caddemi and his coworkers' have conducted research on damage identification of beams by static tests ([4,6–9]), standing out due to the explicit expressions of the parameters to be identified. In particular, in the work of Caddemi and Di Paola [6], a modified version of the Hu-Washizu variational principle was introduced to identify the imperfections in beams, shedding light on the development of a general procedure to obtain closed-form solutions of different identification problems according to the principle.

In this paper, attention is concentrated on the identification of imperfections in thin plates. The static response of a thin plate is governed by a fourth-order partial differential equation, being much more difficult than the beam problem. Alternatively, displacement-based approximate methods, such as the Ritz method, are usually used to acquire approximate analytical solutions, featuring concision and efficiency. However, in the above-mentioned identification procedure based on Hu-Washizu principle, the expressions of internal forces (or stresses) in the elastic body usually need to be assumed, which is not an easy matter for thin plates. Therefore, the modified potential energy principle, with the independent variables of displacements and tractions on the constrained boundary, is used in this paper to obtain displacement-based approximate analytical solutions, avoiding the variables of internal forces.

An improved version of modified potential energy functional is established for the identification purpose, allowing for the response measurements as the additional fictitious constraints. Expressions of the fictitious reactions, as functions of imperfection parameters to be identified, are derived from the stationary conditions of the functional, and nullification of these reactions leads to the identification of the unknown structural parameters.

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The framework proposed in this paper is applicable to different cases of imperfections in thin plates, providing approximate analytical solutions to the identification problems. The procedure is exemplified by three typical applications, showing the generality in a class of inverse problems.

2. The modified potential energy principle

In this section, the modified potential energy principle [22] is introduced as the basic theory for the identification procedure, especially for thin plates. An improved version of the principle is established for the purpose of identification, accounting for the response measurements via experimental tests as fictitious prescribed displacements.

2.1. The modified potential energy principle for linear elasticity

An elastic body in the orthogonal Cartesian coordinate system $x_i\,(i=1,\,2,\,3)$, occupies a domain Ω bounded by the surface S. Denoted by S_u, the constrained part of S has prescribed displacement components $u_i=\bar{u}_i\,(i=1,2,3)$; while the complementary part of S_u, where the tractions are given as $\bar{T}_i\,(i=1,2,3)$, is denoted by S_σ. The well-known principle of minimum potential energy is given as

$$\Pi_\mathrm{p}(u_i)=\int_\Omega \frac{1}{2}D_{ijkl}u_i,_j u_k,_l \mathrm{d}\Omega-\int_\Omega \bar{f}_i u_i \mathrm{d}\Omega \quad -\int_{S_\sigma}\bar{T}_i u_i \mathrm{d}S=\min, \qquad (1)$$

$$\text{subject to} \quad u_i=\bar{u}_i \quad \text{on} \quad S_u,$$

in which $\bar{f}_i\,(i=1,2,3)$ are the assigned body force components, and D_{ijkl}, the Hooke stiffness tensor for isotropic elastic material, is positive definite.

With the Lagrange multiplier method, one can relax the constraints $u_i=\bar{u}_i$ on S_u by introducing Lagrange multipliers and construct an augmented functional, termed the modified potential energy functional

$$\Pi_\mathrm{mp}(u_i,\lambda_i)=\Pi_\mathrm{p}(u_i)-\int_{S_u}\lambda_i(u_i-\bar{u}_i)\mathrm{d}S. \qquad (2)$$

The stationary condition of Π_mp gives

$$\begin{aligned}\delta\Pi_\mathrm{mp}=&-\int_\Omega\left[\left(D_{ijkl}u_k,_l\right),_j+\bar{f}_i\right]\delta u_i\mathrm{d}\Omega\\&+\int_{S_\sigma}\left(D_{ijkl}u_k,_l n_j-\bar{T}_i\right)\delta u_i\mathrm{d}S\\&+\int_{S_u}\left(D_{ijkl}u_k,_l n_j-\lambda_i\right)\delta u_i\mathrm{d}S-\int_{S_u}(u_i-\bar{u}_i)\delta\lambda_i\mathrm{d}S=0,\end{aligned} \qquad (3)$$

where integration by parts and Gauss theorem are applied. In the above derivation, one can identify the Euler equations, i.e. equilibrium equations as,

$$\left(D_{ijkl}u_k,_l\right),_j+\bar{f}_i=0 \quad \text{in} \quad \Omega \quad, \qquad (4)$$

and the boundary conditions as

$$D_{ijkl}u_k,_j n_j=\bar{T}_i \quad \text{on} \quad S_\sigma \quad, \qquad (5)$$

$$u_i=\bar{u}_i\,,\quad \lambda_i=D_{ijkl}u_k,_l n_j \quad \text{on} \quad S_u \quad. \qquad (6)$$

The three Lagrange multipliers are identified as the boundary tractions T_i on S_u, thus the modified potential energy functional can be rewritten as

$$\Pi_\mathrm{mp}\left(u_i,\,T_i\big|_{S_u}\right)=\Pi_\mathrm{p}(u_i)-\int_{S_u}T_i(u_i-\bar{u}_i)\mathrm{d}S \quad, \qquad (7)$$

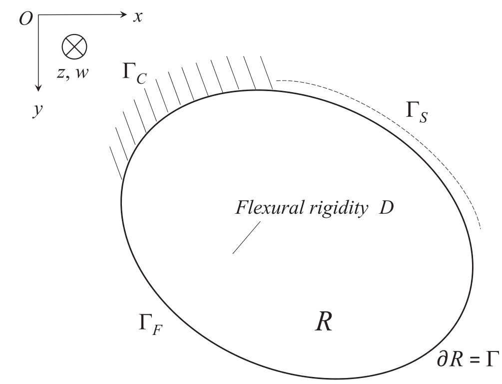

Fig. 1. The thin plate model.

and the modified potential energy principle can be stated as: *The modified potential energy functional* Π_mp *takes stationary value for true* $\left(u_i,\,T_i\big|_{S_u}\right)$.

2.2. The modified potential energy principle for thin plates

Since the present work concentrates on the damage parameter identification of elastic thin plates, the modified potential functional in Eq. (7) should be rewritten in its formulation for thin plates.

Consider an isotropic elastic plate in Cartesian coordinate system $x_i\,(i=1,\,2,\,3,\,\{x_1,\,x_2,\,x_3\}=\{x,\,y,\,z\})$, x_3 is the coordinate perpendicular to the mid-surface of the plate, which occupies the region R bounded with the curve Γ, as shown in Fig. 1. The deflection of the plate along the x_3 axis is denoted by w. On the clamped part of Γ, denoted by Γ_C, the slope angle along the normal direction $w,_n$ is assigned as $w,_n=\bar{w},_n$; while on Γ_C and the simply supported part Γ_S, the deflection is prescribed as $w=\bar{w}$. Besides, normal moment M_n and Kirchhoff shear force V_n are assigned as $M_n=\bar{M}_n$, $V_n=\bar{V}_n$ on the free part Γ_F complementary to $\Gamma_C\cup\Gamma_S$ (i.e. $\Gamma_C\cup\Gamma_S\cup\Gamma_F=\Gamma$), while $M_n=\bar{M}_n$ on Γ_S. Hence, the modified potential energy functional is given as

$$\begin{aligned}&\Pi_\mathrm{mp}\left(w,\,M_n\big|_{\Gamma_C},\,V_n\big|_{\Gamma_C\cup\Gamma_S}\right)\\&=\int_R\frac{D}{2}\left[(1-\mu)w,_{\alpha\beta}w,_{\alpha\beta}+\mu(\nabla^2w)^2\right]\mathrm{d}A\\&\quad-\int_R\bar{q}w\mathrm{d}A-\int_{\Gamma_F}\bar{V}_n w\mathrm{d}\Gamma-\int_{\Gamma_F\cup\Gamma_S}\bar{M}_n w,_n\mathrm{d}\Gamma\\&\quad-\int_{\Gamma_C\cup\Gamma_S}V_n(w-\bar{w})\mathrm{d}\Gamma-\int_{\Gamma_C}M_n(w,_n-\bar{w},_n)\mathrm{d}\Gamma,\end{aligned} \qquad (8)$$

where $\alpha,\beta=1,2$ and q is the distributed load per unit area normal to the plate, and the flexural rigidity is defined as $D=Eh^3/12(1-\mu^2)$ with E, μ being the Young's modulus, Poisson's ratio of the material and h being the thickness of the plate. Moreover, M_n and V_n can be expressed in terms of the deflection w as $M_n=-D\left(\frac{\partial^2 w}{\partial n^2}+\mu\frac{\partial^2 w}{\partial s^2}\right)$ and $V_n=-D\left(\frac{\partial\nabla^2 w}{\partial n}+(1-\mu)\frac{\partial^3 w}{\partial n\partial s^2}\right)$, where n and s indicate the normal and tangent directions at the boundary of the plate.

2.3. The improved version for the purpose of parameter identification

In Eq. (8), the flexural stiffness D can be considered to depend on some structural parameters expressed by a vector $\boldsymbol{\beta}$, i.e. $D=D(\boldsymbol{\beta})$. In the solution to a parameter identification problem, one needs to