Mechanics Research Communications 72 (2016) 41–52

Contents lists available at ScienceDirect

Mechanics Research Communications

journal homepage: www.elsevier.com/locate/mechrescom

Decomposed theorem of a transversely isotropic elastic plate for extensional deformation

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ARTICLE INFO

Article history:
Received 7 July 2015
Received in revised form 9 January 2016
Accepted 11 January 2016
Available online 18 January 2016

Keywords:
Transversely isotropic
Refined theory
Decomposed theorem
Elliott–Lodge solution
Lur'e method

ABSTRACT

Without ad hoc assumptions, a decomposed theorem of a transversely isotropic plate for extensional deformation are derived and studied based on transversely isotropic elastic theory. Firstly, from the Elliott–Lodge solution and Lur'e method, the displacement and stress components are obtained in terms of mid-plane displacements and transverse normal strain. Secondly, the exact equations of the plate are obtained under homogeneous boundary conditions. The general stress state of the plate consists of three parts: the generalized plane-stress state, the shear state, and the Papkovich–Fadle state. At last, the decomposed form of a transversely isotropic elasticity plate for extensional deformation is obtained, and the decomposed theorem is proven strictly for the first time.

1. Introduction

The general deformation of a plate may be decomposed into two parts: anti-symmetric and symmetric [1], and bending deformation and extensional deformation can be gained, respectively. Gao and Zhao [2] investigate plane problems by using the Papkovich–Neuber solution and Lur'e method. Based on Gao and Zhao's work, we extend the refined theory of thick plates to transversely isotropic plates in this paper.

The method utilized to deduce a 2D theory from a 3D theory directly without ad hoc assumptions was originally introduced by Cheng [1] for the development of refined plate theories. A refined plate theory consists of three parts: the bi-harmonic equation, the shear equation, and the transcendental equation. Wang [3–5] extended Cheng's refined theory to transversely isotropic plates and studied plate and plane problems. Gao and Zhao [2] obtained the refined theory of thick plates for extensional deformation by using the Papkovich–Neuber solution and Lur'e method without ad hoc assumptions. Zhao [6] established a refined theory of a transversely isotropic bending plate from the Elliott–Lodge solution and Lur'e method. However, these theories failed to provide strict proof of the decomposed form of a plate. In recent years, in accordance with refined theory of plates, several other scholars extended the theory to transversely isotropic thermoporoelastic beams [7], bi-layer beams for a transversely isotropic body [8], axisymmetric electro-magneto-elastic circular cylinders [9], and so on.

Based on early work, Gregory [10] provided rigorous proof for the decomposed form of isotropic plates. The general stress state of a plate in decomposed theorem consists of three parts: the interior state, the shear state, and the Papkovich–Fadle state. The proof was derived in terms of the Papkovich–Fadle eigenfunction expansion of bi-harmonic functions [11,12]. Wang and Zhao [13] directly and concisely provided new proof that is independent of the Papkovich–Fadle expansion.

Unlike those of plate problems, the equations of extensional deformation problems are well developed in theory and widely utilized for various engineering problems. Hence, we discuss the extensional deformation problems of transversely isotropic plates and derive a decomposed theorem in this study. We provide the decomposed form of a transversely isotropic elasticity plate for extensional deformation and prove the decomposed theorem strictly. In next section, the equations and notations involved are presented. In Section 3, the displacement and stress expressions of the transversely isotropic elasticity plate for extensional deformation are studied with the Elliott–Lodge solution and Lur'e method. The exact plate equations under homogeneous boundary conditions are obtained in Section 4, and the general stress state of the plate consists of three parts: the generalized plane-stress state, the shear state, and the Papkovich–Fadle

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http://dx.doi.org/10.1016/j.mechrescom.2016.01.006
0093-6413/

state. The decomposed form of the plate is obtained, and the decomposed theorem is proven strictly in Section 5. The influence of the Papkovich–Fadle state of the beam is discussed in Section 6.

2. Equations and notations

A homogeneous transversely isotropic plate occupying the domain

$$\Omega = \{(x_1, x_2, x_3) | (x_1, x_2) \in D, |x_3| \le h/2\}, \tag{1}$$

where D is the cross-section of the plate and its thickness is h, the x_3-axis be perpendicular to the isotropic plane of the medium (x_1, x_2) in a Cartesian system (x_1, x_2, x_3).

The constitutive equations for the transversely isotropic body are described to be

$$\begin{cases} \sigma_{11} = C_{11}u_{1,1} + C_{12}u_{2,2} + C_{13}u_{3,3}, \\ \sigma_{22} = C_{12}u_{1,1} + C_{11}u_{2,2} + C_{13}u_{3,3}, \\ \sigma_{33} = C_{13}u_{1,1} + C_{13}u_{2,2} + C_{33}u_{3,3}, \end{cases}$$
$$\begin{cases} \sigma_{23} = C_{44}(u_{2,3} + u_{3,2}), \\ \sigma_{31} = C_{44}(u_{3,1} + u_{1,3}), \\ \sigma_{12} = C_{66}(u_{1,2} + u_{2,1}), \end{cases} \tag{2}$$

where σ_{11}, σ_{22}, σ_{33} are normal stresses, σ_{23}, σ_{31}, σ_{12} are shear stresses, u_1, u_2, u_3 are displacements in the x_1, x_2 and x_3 directions, respectively. C_{ij} are material constants with $C_{66} = (C_{11} - C_{12})/2$. In which the symbol, "$i$" denotes the partial derivative with respect to i.

Let E, G, ν be Young's modulus, shear modules, and Poisson's ratio in the plane of isotropic, respectively; and let E', G', ν' be the transverse Young's modulus, shear modules, and Poisson's ratio, respectively. Then

$$\begin{aligned} C_{11} &= \frac{E(1 - k\nu'^2)}{(1+\nu)(1 - \nu - 2k\nu'^2)}, \\ C_{33} &= \frac{E(1-\nu)}{k(1 - \nu - 2k\nu'^2)}, \\ C_{44} &= \frac{G}{k_g} = \frac{E}{2k_g(1+\nu)}, \\ C_{12} &= \frac{E(\nu + k\nu'^2)}{(1+\nu)(1 - \nu - 2k\nu'^2)}, \\ C_{31} &= \frac{\nu' E}{(1 - \nu - 2k\nu'^2)}, \\ C_{66} &= G = \frac{E}{2(1+\nu)}, \end{aligned} \tag{3}$$

where $k = E/E'$, $k_g = s_0^2 = G/G'$.

The equilibrium equations without body force are

$$\begin{aligned} &\sigma_{11,1} + \sigma_{12,2} + \sigma_{31,3} = 0, \\ &\sigma_{12,1} + \sigma_{22,2} + \sigma_{23,3} = 0, \\ &\sigma_{31,1} + \sigma_{23,2} + \sigma_{33,3} = 0. \end{aligned} \tag{4}$$

The expansion of the strain compatibility equations are

$$\begin{aligned} &\varepsilon_{11,ii} + \varepsilon_{ii,11} - 2\varepsilon_{1i,1i} = 0, \\ &\varepsilon_{22,ii} + \varepsilon_{ii,22} - 2\varepsilon_{2i,2i} = 0, \\ &\varepsilon_{33,ii} + \varepsilon_{ii,33} - 2\varepsilon_{3i,3i} = 0, \\ &\varepsilon_{23,ii} + \varepsilon_{ii,23} - \varepsilon_{2i,3i} - \varepsilon_{3i,2i} = 0, \\ &\varepsilon_{31,ii} + \varepsilon_{ii,31} - \varepsilon_{3i,1i} - \varepsilon_{1i,3i} = 0, \\ &\varepsilon_{12,ii} + \varepsilon_{ii,12} - \varepsilon_{1i,2i} - \varepsilon_{2i,1i} = 0, \end{aligned} \tag{5}$$

where the similar subscripts express the sum of 1 to 3. And ε_{11}, ε_{22}, ε_{33} are normal strains, ε_{23}, ε_{31}, ε_{12} are shear strains.