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On the interplay between material flaws and dynamic necking

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ABSTRACT

In this paper we investigate the interplay between material defects and flow localization in elastoplastic bars subjected to dynamic tension. For that task, we have developed a 1D finite difference scheme within a large deformation framework in which the material is modelled using rate-dependent J_2 plasticity. A perturbation of the initial yield stress is introduced in each node of the finite difference mesh to model localized material flaws. Numerical computations are carried out within a wide spectrum of strain rates ranging from 500 s^{-1} to 2500 s^{-1}. On the one hand, our calculations reveal the effect of the material defects in the necking process. On the other hand, our results show that the necking inception, instead of being a random type process, is the deterministic result of the interplay between the mechanical behaviour of the material and the boundary conditions. This conclusion agrees with the experimental evidence reported by Rittel et al. [1] and Rotbaum et al. [2].

1. Introduction

Whether dynamic necking in elastoplastic solids is a random or deterministic process remains as an open question. This issue, which has triggered critical debates in the Solid Mechanics community during the last decade, has been typically addressed using two different uniaxial tensile configurations [3].

On the one hand, we have the radial expansion of ductile rings [4–6]. The geometric and loading symmetries of this problem nearly eliminate the effects of wave propagation before the onset of necking, which reveals the *true* mechanical properties of the material. For years, it was accepted that the multiple localization pattern which precedes fragmentation in the ring expansion test is a random process, controlled to a large extent by geometric and material defects. However, some recent publications [7,8] have raised the possibility that the localization process becomes deterministic for sufficiently high expansion velocities. The increase of the inertia forces with the loading rate helps to regularize the problem and promotes the emergence of uniform necking patterns which reveal the deterministic nature of the localization process.

On the other hand, we have the impact testing of *linear* tensile specimens [9,10]. The sample, initially at rest, is subjected to a sudden axial velocity which, unlike what happens in the ring expansion test, leads to the generation of stress waves. It has been frequently assumed in the literature that, despite the stress waves intervention within the specimen, the necking inception in the impact tensile test is a random process. However, recent experimental works [1,2] suggested that flow localization in the dynamic tensile test is the deterministic result of the interplay between material behaviour, specimen geometry and boundary conditions. In this paper we develop a numerical methodology which supports such experimental finding. We carry out computations using a finite difference model in which material flaws are included. Our results indicate that wave propagation phenomena control, to a large extent, flow localization in elastoplastic specimens subjected to impact tensile loading.

2. Constitutive equations

The material behaviour is described by a hypoelastic-plastic constitutive model which follows the standard principles of Huber-Mises plasticity.

The evolution equation for the Kirchhoff stress $\boldsymbol{\tau}$ is:

$$\boldsymbol{\tau}^{\nabla} = \mathcal{L} : \mathbf{d}^{e} \quad (1)$$

where $\boldsymbol{\tau}^{\nabla}$ is the Green–Naghdi objective derivative of the Kirchhoff stress tensor. We have followed the works of Holzapfel [11] and Sumelka [12], and use the Kirchhoff stress in the formulation of the constitutive equations. This is considered the most directly available stress measure when an elastic reference state is considered. Moreover, the fourth order isotropic elasticity tensor $\mathcal{L}$ and the elastic rate of deformation tensor $\mathbf{d}^{e}$ are defined as follows:

$$\mathcal{L} = 2G\mathcal{I} + \lambda \mathbf{I} \otimes \mathbf{I} \quad (2)$$

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Table 1
Physical constants, elastic parameters and parameters related to the yield stress for AISI 430 steel [13].

Symbol	Property and units	Value
ρ_o	Initial density (kg/m^3), Eqs. (13) and (14)	7740
C_p	Specific heat (J/kg K), Eqs. (15) and (28)	460
k	Thermal conductivity (W/m K), Eqs. (15) and (28)	26.1
G	Lamé's constant (GPa), Eqs. (2), (25), (26) and (27)	75.2
λ	Lamé's constant (GPa), Eq. (2) and (26)	146
E	Young's modulus (GPa), Eq. (23)	200
ν	Poisson's ratio	0.33
A	Initial yield stress (MPa), Eq. (6)	175.67
B	Work hardening modulus (MPa), Eq. (6)	530.13
h	Work hardening exponent, Eq. (6)	0.167
$\dot{\varepsilon}_{ref}$	Reference strain rate (s^{-1}), Eq. (6)	0.01
m	Strain rate sensitivity exponent, Eq. (6)	0.0118
T_{ref}	Reference temperature (K), Eq. (6)	300
μ	Temperature sensitivity exponent, Eq. (6)	0.51
β	Taylor-Quinney coefficient, Eqs. (15) and (28)	0.9

$$\mathbf{d}^e = \mathbf{d} - \mathbf{d}^p \quad (3)$$

where G and λ are the Lamé's constants, $\mathcal{I}$ is the fourth order identity tensor and $\mathbf{I}$ is the second order identity tensor. $\mathbf{d}$ and $\mathbf{d}^p$ are the total and plastic rate of deformation tensors, respectively.

The yield function f is written as:

$$f = \bar{\tau} - \sigma_Y = 0 \quad (4)$$

where the equivalent stress $\bar{\tau}$ and the yield stress σ_Y are defined as follows:

$$\bar{\tau} = \sqrt{\frac{3}{2}(\mathbf{s} : \mathbf{s})} \quad (5)$$

$$\sigma_Y = A + B\left(\bar{\varepsilon}^p\right)^h \left(\frac{\dot{\bar{\varepsilon}}^p}{\dot{\varepsilon}_{ref}}\right)^m \left(\frac{T}{T_{ref}}\right)^{-\mu} \quad (6)$$

where $\mathbf{s} = \boldsymbol{\tau} - \frac{1}{3}(\boldsymbol{\tau} : \mathbf{I})\mathbf{I}$ is the deviatoric part of the Kirchhoff stress tensor, $\dot{\bar{\varepsilon}}^p = \sqrt{\frac{2}{3}\left(\mathbf{d}^p : \mathbf{d}^p\right)}$ is the equivalent plastic strain rate, $\bar{\varepsilon}^p = \int_0^t \dot{\bar{\varepsilon}}^p(\xi)\mathrm{d}\xi$ is the equivalent plastic strain and T is the current material temperature. A, B, h, m and μ are material parameters while $\dot{\varepsilon}_{ref}$ and T_{ref} are the reference strain rate and temperature.

The flow rule is given by:

$$\mathbf{d}^p = \frac{\partial f}{\partial \boldsymbol{\tau}}\dot{\bar{\varepsilon}}^p = \frac{3}{2}\frac{\mathbf{s}}{\bar{\tau}}\dot{\bar{\varepsilon}}^p \quad (7)$$

The formulation of the constitutive model is completed by introducing the Kuhn–Tucker loading/unloading complementary conditions:

$$\dot{\bar{\varepsilon}}^p \geq 0, \quad f \leq 0, \quad \dot{\bar{\varepsilon}}^p f = 0 \quad (8)$$

and the consistency condition during plastic loading:

$$\dot{f} = 0 \quad (9)$$

In the calculations of this paper we use the constitutive parameters corresponding to the AISI 430 steel [13]. These are given in Table 1.

3. Governing equations

We consider a cylindrical bar of initial length L = 6 mm and cross-section diameter Φ = 3 mm subjected to dynamic stretching. Within a 1D analysis we do not need to specify the shape of the cross-section area, nevertheless we consider that the bar is cylindrical (to calculate the force in the sample, see Fig. 2), as a typical tensile specimen. The dimensions of the bar correspond to samples used in Split Hopkinson Tensile Bar experiments [2,14]. The problem is posed in one-dimensional form.

The relation between the Eulerian z and the Lagrangian coordinate $Z\,(0 \leq Z \leq L)$ is given by:

$$z = Z + U_Z \quad (10)$$

where U_Z is the displacement along the axial direction. The logarithmic strain ε_Z and strain rate $\dot{\varepsilon}_Z$ along the axial direction are given by:

$$\varepsilon_Z = \ln(1 + \partial U_Z/\partial Z) \quad (11)$$

$$\dot{\varepsilon}_Z = \partial \varepsilon_Z/\partial t \quad (12)$$

The fundamental equations, formulated in Lagrangian coordinates, which govern the loading process are given below.

- Mass conservation:

$$\rho_0 = \rho \mathrm{J} \quad (13)$$

where ρ_0 is the initial material density, ρ is the current density and $\mathrm{J} = e^{(1-2\gamma)\varepsilon_Z}$ is the Jacobian determinant of the deformation gradient tensor, where γ is a material parameter specified in Section 4.

- Momentum balance in the axial direction:

$$\rho_0 \Lambda_0 \frac{\partial^2 U_Z}{\partial t^2} = \frac{\partial}{\partial Z}\left(\frac{\Lambda}{\mathrm{J}}\tau_Z\right) \quad (14)$$

where Λ_0 and Λ are the initial and current cross-section areas of the bar and τ_Z is the Kirchhoff stress along the axial direction.

- Conservation of energy:

$$\rho C_p \frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial Z^2} + \beta \tau_Z \dot{\varepsilon}_Z^p \quad (15)$$

where C_p is the specific heat, k is the conductivity, β is the Taylor-Quinney coefficient and $\dot{\varepsilon}_Z^p$ is the plastic strain rate along the axial direction. The thermoelastic effects are neglected. Note that, for the sake of simplicity, the spatial derivative which appears in the conductivity term is taken as a Lagrangian derivative (small strains in the conductivity term).

- Stress rate: the Green–Naghdi objective derivative, due to the one-dimensional nature of the model, is computed as a simple time derivative:

$$\tau_Z^{\nabla} = \dot{\tau}_Z \quad (16)$$

Considering the domain $[0, L]$, Eqs. (13)-(16) are numerically solved under the following initial and boundary conditions formulated in Lagrangian coordinates:

$$U_Z(Z, 0) = 0; \quad \tau_Z(Z, 0) = 0; \quad T(Z, 0) = T_0$$

$$U_Z(0, t) = 0; \quad V_Z(L, t) = V^{imp}; \quad \frac{\partial T(0, t)}{\partial Z} = \frac{\partial T(L, t)}{\partial Z} = 0$$

where T_0 is the initial temperature taken as 300 K and V^{imp} is the impact velocity.

4. Finite difference model

Following the works of Zhou et al. [6], Regazzoni et al. [15] and Ravi-Chandar and Triantafyllidis [16], we have developed a 1D finite difference model to describe the mechanical behaviour of elastoplastic bars subjected to dynamic tension. This numerical approach was presented and validated with experiments and finite element calculations in our previous work [17]. In the present paper we only show the main features of the model for completeness.