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Rotation control of a parametrically excited pendulum by adjusting its length

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ABSTRACT

We present a new control strategy for the vertically excited parametric pendulum, with a view on energy harvesting from rotating motion. Two possible energy sources are considered: a vibrating machine, represented by a sinusoidal excitation; and the sea waves, simulated by a stochastic process. In both cases, rotations can be achieved and maintained only for some forcing conditions. Thus, as stable rotations are required, the pendulum must be controlled. We propose to perform this control by means of a telescopic adjustment of the pendulum length during the motion. The idea is to give the pendulum an aid to reach and maintain rotations, accelerating the motion by modifying conveniently the position of the mass. To a better understanding of this concept, one may think of a child on a swing, who extends or retracts his legs in order to accelerate the motion. Numerical simulations show that, with a control action of this kind, stable rotations can be reached regardless of the forcing conditions and for every set of initial conditions. These are very promising results in terms of energy harvesting, since an optimized application of this control technique can lead to the design of autonomous pendulum harvester devices.

1. Introduction

The increasing global awareness about the environmental damage has prompted in recent years the search for clean energy sources. The vast energy availability in ambient vibrations allows the development of many systems, aimed at the recovery of energy from variety of sources. These sources include the motion of large bodies of water [1,2], the biomechanics of a walking person [3] and the vibrations in civil structures [4], industrial machines [5] or flying planes [6], among others [7–9].

The vertical parametric pendulum was firstly proposed as an energy harvester device by Prof. Marian Wiercigroch [2]. Since the beginning of the century, he and his co-workers, and also other scientists, explored the ability of such systems for energy extraction from the ocean waves [10–16]. The idea consists of a pendulum on a floating platform, which is in turn vertically excited by the waves at a given (averaged) frequency. If stable rotational motion is achieved, a generator attached to the pendulum axis may produce electrical energy [12]. Although the idea is very simple and intuitive, its implementation is not trivial since stable rotations may be difficult to obtain, even with a simple sinusoidal excitation [17]. This is due to a strong dependence on forcing parameters and initial conditions of position and velocity. Thus, to reach stable rotations, a control strategy is necessary [11].

One of the forcing conditions we consider in this work is a sinusoidal excitation, with constant forcing parameters of amplitude and frequency. This situation can represent the motion imposed to the pendulum by an industrial vibrating machine. In this case, stable pure rotations (i.e. no oscillating behavior of any kind, [18]) can be reached only for some combinations of the forcing parameters [17]. But even if these parameters can be conveniently chosen, rotations coexist with other responses such as oscillations and chaotic motion, depending on the initial conditions [19]. Moreover, it has been demonstrated [12] that, in practical terms, stable pure rotations are possible only in a subset of the theoretical region of the parameter space. Being the forcing parameters constant, part of the problem can be solved with an adequate design, by tuning the system within the region of the parameter space in which rotations are possible. With a good design, control of rotations is required only to correct responses due to unsuitable initial conditions.

The other excitation considered is the motion of the sea waves, which is simulated as a stochastic process [11]. In this situation,

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there are multiple forcing parameters of amplitude and frequency (determined by a spectral density), and there is not a suitable design for tuning the system. The response of the pendulum is also stochastic, and active control is necessary not only to ensure rotational response irrespective of initial conditions, but also to deal with a continuously changing load.

We propose an active control by means of an actuator, which adjusts the pendulum length during the motion. This idea has been successfully applied for a simple damped pendulum [20]. The telescopic adjustment gives the pendulum an aid to reach and maintain rotations, accelerating the motion by modifying the position of the mass according to a convenient strategy. The control function proposed to govern the change of length uses as input the measurement of the angle and velocity of the pendulum. Since rotations produce arbitrary large angle values, a sine function is used in order to keep the rotation control as dependent on the angular position but not on the actual angle measurement. The dependence on angular velocity is modeled with a logistic function, which value depends on whether velocity adopts positive or negative values. The application of the control is limited by a predefined threshold velocity, and steepness factors are defined to get a realistic value for the actuator velocity.

The article is organized as follows. After this introduction (Section 1), the equation of motion of the vertical parametric pendulum with variable length is derived, considering sinusoidal and stochastic excitations (Section 2). Then, the control strategy employed to adjust the pendulum length is presented and explained (Section 3). Finally, the results of numerical simulations comparing controlled and uncontrolled pendulums are presented and discussed (Section 4).

2. The vertical parametric pendulum with variable length

2.1. Equation of motion

Consider the pendulum system given in Fig. 1, which axis of rotation is excited by an imposed motion $Y = Y(t)$. The total length of the pendulum, l, can be modified by adjusting the position of the telescopic rod, of length l_2. When this rod is retracted, the pendulum recovers its natural length l_1 ($l = l_1$). The equation of motion of this system can be set up by using Lagrange's equation for single degree-of-freedom non-conservative systems, which can be expressed as.

$$\frac{d}{dt}\left(\frac{dT}{d\dot{\theta}}\right) - \frac{dT}{d\theta} + \frac{dV}{d\theta} + \frac{dD}{d\dot{\theta}} = 0 \tag{1}$$

where θ is the angle measured from the downward hanging position (positive in counter-clockwise direction), while T, V and D

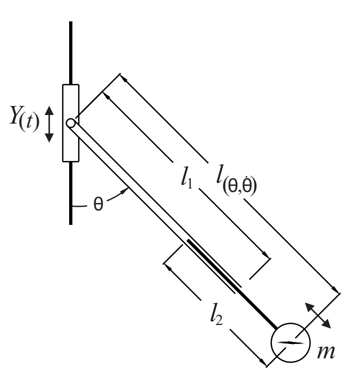

Fig. 1. Scheme of the vertical parametric pendulum with variable length.

represent kinetic, potential and dissipative energy of the system, respectively. These energy magnitudes are

$$\begin{aligned} T &= \frac{m}{2}[(l\dot{\theta}\cos\theta)^2 + (\dot{Y} + l\dot{\theta}\sin\theta)^2] \\ V &= mgl(1-\cos\theta) \\ D &= \frac{1}{2}c_\theta\, l^2\dot{\theta}^2 \end{aligned} \tag{2}$$

where m is the mass of the bob, c_θ is the viscous damping coefficient and g the acceleration of gravity.

Introducing Eq. (2) into Eq. (1) and performing the corresponding derivatives, the equation of motion of the system can be obtained as

$$ml^2\ddot{\theta} + m(l\ddot{Y} + lg + \dot{l}\dot{Y})\sin\theta + (c_\theta l^2 + 2ml\dot{l})\dot{\theta} = 0 \tag{3}$$

where $\dot{l}$ represents physically the linear velocity of the telescopic actuator. Eq. (3) allows studying the dynamics of the pendulum for an arbitrary time-dependant imposed motion Y.

2.2. Imposed motion as a sinusoidal wave

If the imposed motion is a wave of the form $Y(t) = -A\cos(\Omega t)$, Eq. (3) takes the form (after dividing by ml^2)

$$\ddot{\theta} + \left(\frac{c_\theta}{m} + 2\frac{\dot{l}}{l}\right)\dot{\theta} + \frac{1}{l}\left[A\Omega^2\cos(\Omega t) + g + \frac{\dot{l}}{l}A\Omega\sin(\Omega t)\right]\sin\theta = 0 \tag{4}$$

For l constant, Eq. (4) recovers the classical equation of motion of the vertical parametric pendulum [17,21].

2.3. Imposed motion as a simulated sea wave

A sea wave can be represented by a composition of sinusoidal waves with frequencies determined by a spectral density. To simulate the time history of a wave, we use the approach presented in reference [11], based on the Shinozuka–Jan method for random processes [22] and the Pierson–Moskowitz spectral representation of the sea waves [23]. This power spectral density is given by

$$S(\Omega) = \frac{8.1g^2}{10^3\Omega^5}\exp\left(-0.032\frac{g^2}{\Omega^4 H_s^2}\right) \tag{5}$$

where H_s is the significant wave height, corresponding to 1/3 of the highest wave measured. With the statistical information of Eq. (5), the stochastic time-dependant excitation of the sea wave is expressed as

$$Y(t) = \sqrt{2}\sum_{i=1}^{N}\sqrt{(\Omega_i - \Omega_{i-1})S(\Omega_i)}\cos(\Omega_i t + \phi_i) \tag{6}$$

where ϕ_i are random phase angles and N is the number of sampled frequencies. The frequency range, $[\Omega_0, \Omega_N]$ must be set in a way that most of the spectrum S to be contained at that range. The frequency intervals, $\Omega_i - \Omega_{i-1}$, are determined by solving the following equation

$$\int_{i-1}^{i} S(\Omega)d\Omega = \int_{i}^{i+1} S(\Omega)d\Omega \tag{7}$$

The fulfillment of Eq. (7) ensures that the area under the curve of $S(\Omega)$ is the same for all frequency intervals, with the consequent good covering of the highest spectral density zone. The wave model of Eq. (6) was tested against real data, producing a good agreement [11]. Considering Eq. (6) as the imposed motion required in Eq. (3), we can study the behavior of the pendulum with adjustable length under the excitation of the sea waves.